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AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

18th Annual Convention.

New York and Buffalo, August 14–24, 1901.

New York, Aug. 14, 1901.

The invitations extended to European electrical engineers to attend the 18th Annual Convention of the Institute at Buffalo, beginning August 20th, were responded to in person by the following gentlemen: Prof. Paul Janet, Director of the Laboratoire Central and of L'Ecole Superieur de l'Electricité, official delegate of the Société Internationale des Electriciens; M. Ch. David Paris; M. J. Courbier, Laboratoire Central de l'Electricité; M. Boy de la Tour, Chief Engineer of Société Fivés-Lille; M. Devauve, standards department Maison Breguet, Paris; M. Marcel Aubert, Ingenieur, Licencié es Sciences; M. Maurice Aubert, Ingenieur, Licencié es Sciences; M. Roux, Ingenieur Civil. All of the above gentlemen are members of the Société Internationale des Electriciens. Herr. F. W. Ch. Janisch, Chief Engineer Siemens & Halske Company, Berlin; Prof. Gustave Gillon, University de Louvain, Belgium; Mr. D. McNaught, Silvertown, England; Mr. Norman C. Sawers, Silvertown, England; Mr. Arthur H. Potts, London, England; Mr. G. H. Cottam, London; Mr. John Denham, Capetown; Mr. A. T. Cooper, Reading, England; Mr. R. L. Gamlin, Kent, England.

As announced in the program, a formal welcome was extended to the foreign guests of the Institute at the House of the American Society of Mechanical Engineers, 12 West 31st Street, New York City, on August 14th, 1901. President Steinmetz, in calling the meeting to order, expressed his great pleasure in meeting the European engineers who had accepted the invitation to visit the country, and attend the Annual Convention. He invited Chairman Mailloux of the Council Committee to out-

line the features of the excursions and technical visits which had been arranged for. Upon the conclusion of Mr. Mailloux's remarks, M. Janet, in behalf of the French delegation, warmly expressed in his native tongue, the pleasure experienced by his associates and himself in being able to participate in the proceedings of the INSTITUTE. The members and guests having registered, luncheon was announced, which terminated the proceedings for the day.

On Thursday, August 15th, by invitation of the Crocker-Wheeler Company, the works at Ampere were visited, transportation by special cars being provided by that company. The guests were welcomed upon arrival by President Wheeler and Mr. Dunn. The party returned from Ampere by the 11.43 A. M. train, and reassembled at the Everett House, New York City, where luncheon was arranged for by Chairman Mullin of the Finance Committee. In the afternoon by invitation of the New York Telephone Company, the telephone exchange at 18th street and Irving Place was visited. In this vicinity the excavating for the Rapid Transit tunnel was also inspected.

The International Convention Banquet was held in the evening at the Hotel Marlborough, ninety members and guests with many ladies being present. Past-President Hering presided in the absence of President Steinmetz, while Past-President Martin officiated as Toastmaster. The toasts were responded to as follows: "The Land of Ampere," by M. Janet; "The Countrymen of Faraday," by Mr. N. C. Sawers; "The Successors of Ohm," by Prof. W. Hallock; "The Heirs of Volta," by Mr. Philip Torchio, and "The Legatees of Franklin," by Dr. Schuyler S. Wheeler.

On Friday, August 16th, the party reassembled at the new Edison power-house, foot of 38th Street at the East River. Under the direction of Mr. Calvin W. Rice printed leaflets had been provided giving the technical data of the different power stations which were to be visited. Experienced guides were also at hand in order to give all possible information to the visitors. Through the courtesy of the Brooklyn Edison Company, the New York Edison Company, the Manhattan Railway Company, and the Metropolitan Street Railway Company, the special steamboat "Crystal Stream" had been chartered to convey the party between the different power stations fronting on the East River, and after leaving 38th Street, stops were made at the 76th Street power-house of the Manhattan Railway Company, the 96th Street power-house of the Metropolitan Street Railway Company, the Gold Street station, and the Bay Ridge station of the Brooklyn Edison Company. A bountiful collation was served on board of the boat, while she made a detour around Blackwell's Island in order to permit a more extended view of the approach to New York City from Long Island Sound. For the better accommodation of all, a landing was made at the

Battery upon the return trip. A party of guests under the guidance of Chairman Mullin and Past-President Hering, went from Bay Ridge to Manhattan Beach for the evening.

No special assignments had been made for Saturday, August 17th, it being necessary that personal preparations should be made for the Buffalo trip.

On August 18th, the steamboat "Montauk," which had been chartered by the Transportation Committee, awaited the party at the foot of 43d Street, North River. The outlook was gloomy as a brisk rain was falling at 9 A. M., the hour fixed for departure. It was half-past nine when the boat started, with about 140 people on board. The rain soon ceased, and although the sun failed to shine, the air was clear, and the trip up the Hudson was most enjoyable, the cuisine as usual being carefully looked out for. Upon arrival at Albany about eight o'clock P. M., special electric cars conveyed the party to the Hotel Ten Eyck, where all were made comfortable for the night. Monday morning at half-past eight o'clock, the party boarded the special train, which pulled out of the Union Station over the New York Central Railway. The train landed its passengers inside the yard of the General Electric Works at Schenectady, and every opportunity was afforded the visitors for a thorough examination of this great establishment. An efficient corps of guides had been provided, and no restrictions were enforced. Carriages had been provided for the entertainment of the ladies, and while they were being driven about the city and vicinity, the gentlemen made a tour of the factory. At noon the entire party reassembled in one of the new shops, where a fine luncheon was served, presided over by Vice-President Rice of the General Electric Co. At one-thirty P. M. the party again boarded the train, which was in waiting, and a quick run was made to Buffalo, that city being reached about eight P. M. Most of the party went to the Niagara Hotel, which had been selected as the headquarters during the Convention.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Eighteenth Annual Convention.

Buffalo, N. Y., August 20, 21, 22, 23, 24, 1901.

Tuesday, August 20.

President Charles P. Steinmetz called the Convention to order at 11 A.M., in the Assembly Room of the New York State Building, at the Exposition grounds.

President Steinmetz:—As President of the American Institute of Electrical Engineers, I herewith open the second session of the Annual Convention. Accepting the very kind invitation extended to us, we have assembled here at the New York State Building, on the grounds of the Pan-American Exposition, to carry on our work, consisting in the reading and discussion of papers, in scientific and social conversation, and incidentally to take part in some entertainment.

Before proceeding with any regular program I will call for a few words from Prof. George F. Sever, who has been so intimately connected with the success of the Pan-American Exposition and its electrical features, which, as we all know, are the most prominent and the most important at the Exposition.

Prof. George F. Sever:—Mr. President and members of The American Institute of Electrical Engineers and their distinguished guests, I bid you a hearty welcome to the Pan-American Exposition. It is certainly very gratifying to see this distinguished body of electrical engineers present at this magnificent Exposition, and I can only say: Here it is. Take it. Investigate it. Most of it, of course, is nailed down, but the illumination you can carry away in your minds and forever call your own. The buildings devoted to the exhibits are by no means as large as were those at Chicago and at various other expositions, but we feel that they contain articles of great merit, and illustrating the very latest advances in all lines of art, industry

and science. Those of you who have seen the illumination of the buildings and grounds and the other details, certainly can appreciate the magnificent work that has been done by the engineers connected with the Exposition.

I hope you will make free use of the engineers of the Exposition so far as they can be of service to you in the giving of information. We are perfectly ready and willing to do what we can. The electrical exhibitors in the Electricity Building have placed their services at your disposal in order to guide you about the grounds and to show you details which you might find it difficult to learn about. The office of the Superintendent of the Electrical Exhibits you can count upon as your headquarters in case you desire any knowledge or information pertaining to the Exposition.

I appreciate the opportunity of appearing before you and welcoming you to this magnificent Exposition, and I trust that your stay here will be most pleasant and fully up to the expectations which you have had in mind.

PRESIDENT STEINMETZ:—I believe I voice your sentiments in thanking Prof. Sever heartily for the kind welcome he has extended to us, and I have not the least doubt but that his hopes will be realized; that we shall feel very gratified and very satisfied with all that we see here, and not only that, but that we shall carry away with us pleasant recollections and the benefit of a decided advance in our intellectual and scientific development.

The Secretary made some announcements, and the Convention then adjourned to August 21st.

MORNING SESSION, WEDNESDAY, AUGUST 22ND, 1901.

[For P. M. Lincoln's and W. H. Browne's papers see June and July issue. Caryl D. Haskins' and H. P. Davis's papers follow.]

PRESIDENT STEINMETZ:—Gentlemen, regarding the discussion it has been decided that we shall not have separate discussions of each paper, but first read all the papers dealing with similar subjects, and then have a discussion of them jointly. In pursuance of this arrangement I now call on Mr. Caryl D. Haskins to read his paper on "Some Fundamentals of Electric Meters." I may add that one of the papers which was intended for to-morrow, the paper on "Power Factor Indicators," by Mr. W. H. Browne, Jr., will be read to-day, as dealing with a subject very closely allied to to-day's subject.

A paper presented at the 10th Annual Convention of the American Institute of Electrical Engineers, Buffalo, August 21st, 1901 President Steinmetz in the Chair.

SOME FUNDAMENTALS OF ELECTRIC METERS.

BY CARYL D. HASKINS.

In preparing for presentation to the INSTITUTE my third paper on the general subject of recording meters, I have found it somewhat difficult to sift out from the vast mass of details a very few of the most vital considerations in construction and operation, which seem chiefly to merit consideration and discussion at your hands. It is the mass of individually trivial details which to-day constitute the chief field of study and investigation in electric metering.

The broader and more basic characteristics of the art have been widely discussed long since. Like the whole field of electrical engineering, recording meter design has progressed rapidly. A very few years ago our attention was given to such general issues as how to make a meter that would register on a dial, or how to make a meter that would not necessitate the winding of a clock, or how to make a meter that would record with tolerable accuracy through a range somewhat better than 4 to 1. To-day the art has reduced itself to what might be termed "A microscopic study of trivialities" purposed to gain such improvements as a few years since would have been regarded as academic; as, for example, better behavior on variations of wave form, an increase of 1% or less in average accuracy on loads of from 1% to 5% of the meter's rating, the maintenance of straight line accuracy on loads having power factors substantially unheard of commercially a few years ago, and the like.

This last consideration, that of accuracy on inductive loads, is one of the greatest importance, and has probably not received as much attention at the hands of meter engineers as it merits. I have not chosen to deal with it to-day, because it has been my desire in preparing this paper to lay a very few essentials before you in a thoroughly analytical manner.

To properly deal with meter accuracy under inductive loads, a paper should be devoted to this subject alone, and the subject would necessarily confine one to the consideration of alternating current meters only, a limitation which I by no means wish to accept.

Accuracy on inductive loads is unquestionably an absolute essential for all meters destined for use on alternating circuits carrying any other loads than incandescent lamps, but the elaboration of this topic before you to-day belongs more properly to my colleagues than to me.

I have chosen for my theme perhaps the most important of the considerations which are to-day engrossing the attention of meter engineers—namely, the relation between torque, friction, aging of various kinds and local surroundings. These four factors constitute the chief governing conditions in connection with meter accuracy, on loads of less than 20% of the meter's average rating. These considerations are, of course, confined to the typical motor meter. We have, however, long since come to regard the motor type of meter as the only electrical integrating device capable of being manufactured at a cost rendering it a commercial possibility, whilst retaining the degrees of reliability, convenience and accuracy demanded by the American market. It may indeed be fairly said that the considerations I have selected are the dominant ones in the American meter art. Substantially all of the motor meters known to the American market have a straight line law within their limitations, if friction be eliminated from consideration.

The limitations are, I admit, somewhat severe, but I am probably entirely safe in saying that the electricity meter, as the American purchaser receives it, is subject to a straight line law on non-inductive loads of the frequency, wave form and potential for which it has been calibrated.

Unfortunately friction cannot be eliminated, but its effect on accuracy may be minimized to a remarkable degree.

There is relatively little difficulty in producing a recording meter in which the effect of friction on accuracy is negligible,

even to a degree which may defy detection, at the time the meter issues from the laboratory. But friction causes are manifold and elusive, and I doubt if I overstate the mechanical situation when I say that a meter having the least evidence of friction in its initial condition may, at the end of a period of use, show more error due to friction and consequent light load error than other meters which had much more initial friction.

This is the abstract problem which lies before the engineer, and it has been met by numerous alternatives, all more or less effective.

Taking the next step in the analysis of the situation, we note that the motor element of a recording meter does two kinds of work: First, the driving of a dynamo in the form of a foucault current disk, or the equivalent—which is the work contemplated in the meter's theoretic law. The ratio of this work does not vary, provided the permanent magnets are literally permanent, except under physical conditions which can easily be provided for and guarded against. Second, friction work not contemplated in the theoretic law, and therefore tending to distort the accuracy curve.

Friction is relatively unobjectionable so long as it remains a constant value, for as a constant value it can be readily compensated. The meter having X friction can scarcely be regarded as better than the meter having 10 X friction, provided these values do not change with time or load, a condition which we cannot unfortunately find in practice.

In seeking the alternative remedies for this condition three courses of study present themselves to us: The minimizing of friction as an individual study, means for increasing friction compensation with increase of friction, and finally, the radical increase of foucault current load in relation to possible friction load.

The two latter alternatives have always presented themselves to me as the most promising.

The dissipation of a relatively very large amount of energy in the foucault current disk renders wide variations of friction substantially negligible. In other words, if we can make the work contemplated in the law of the meter a large value, then the variations of work not contemplated in the law become insignificant in relation to the whole work. This means that we must dissipate in the foucault current disk the largest possible

amount of energy consistent with economy. This statement, translated into practical terms, means permanent magnets of the greatest possible strength and a foucault current disk of the greatest possible conductivity consistent with the physical limitations.

The whole essence of success in a motor meter necessarily lies in literally permanent magnets. Since it is vastly easier to make a weak permanent magnet than a strong one, the temptation to get along without the dissipation of a maximum amount of energy in the disk is very great.

If one could make only very weak damping magnets, and therefore dissipate in the foucault disk a very small amount of energy, it would naturally become necessary to get initial friction very low to secure initial accuracy, and in doing this there is always grave danger of sacrificing a permanent minimum of friction to a temporary minimum of friction.

At this point the meter engineer must, if he is to follow the right path, become a physicist, a metallurgist, and, in some measure, a chemist; for he must make a deep and consistent study of strong permanent magnets, which involves a careful investigation of the chemical and physical properties of steel, and of the best means for introducing the initial magnetism, and of reducing this initial magnetism to the largest permanent residue that the magnet can be relied upon to retain. The shaping of the permanent magnets is also of the very greatest importance. How important you probably realize, for this subject has already been carefully dealt with in a most able essay by Dr. Kennelly.

Certain recording meters use a copper foucault disk in place of the more common aluminium disk. With a copper disk of a given thickness, moving at a given speed, more work can be expended in the generation of foucault currents with a magnet of given strength than with any other metal commercially possible. To substitute an aluminium disk having the same peripheral speed and the same thickness, means either to decrease the total work in the disk or to increase the strength of the permanent magnets. The latter is a desirable alternative which can possibly be achieved. The former is a highly undesirable alternative, for reasons which I have stated.

On the other hand, to substitute an aluminium disk of the same conductivity and purposed to rotate at the same speed as

the copper disk, it is necessary to increase the thickness of the disk. This means that the jaws of the magnets must be further separated, if the clearances have already been reduced to a minimum. This, in its turn, means less density in the permanent magnets and less drag or load.

I have mentioned these tributary considerations to show the conditions which surround the work of the engineer whose purpose is to design a meter which shall expend a maximum proportion of total work in the foucault disk, and to show the chief danger in this direction of effort—namely, that of resorting to a magnet which may not be permanent.

Having achieved a meter mechanism in which the work expended in rotating the foucault disk is relatively high, and having constructed a mechanism in which the *changes* of friction are a minimum, rather than a mechanism in which *initial* friction is a minimum, it is next necessary to provide means for immediately offsetting external conditions, which exert a modifying effect upon the friction value.

In the earlier days of meter operation it was regarded as desirable to reduce friction periodically to as nearly its initial value as possible by cleaning and the like.

It may be regarded as an axiom that friction will increase with use. This increase may be large or small in relation to the whole friction dependent upon design, and it may be large or small in relation to the total work of the meter, dependent largely upon the amount of work expended in the foucault disk.

All friction, however, as well as friction increases, is tributary to and governed by the conditions surrounding installation, and the chief of these external influences is vibration, though the literal results of time, consequent corrosion, etc., as well as changes of temperature, play their part.

The effect of vibration on the work value of friction is great, for perfectly obvious physical reasons, and this effect of vibration on friction becomes more prominent as the friction value increases in relation to total work.

It is, therefore, not sufficient that friction should be neutralized by compensation at an average point of calibration. It is important that ready means for variation of friction compensation should be provided, which can be instantly adjusted to meet the actual conditions surrounding the meter at its point of use. Such means for local adjustment have been provided in numerous

forms, and are, I venture to think, highly necessary to the ideal electric meter.

Granting that the variations of friction load due to local surroundings can be accurately and quickly compensated by adjustment of the strength of friction compensation, every meter can be made substantially perfect for the point where it is actually to be used, without in any way affecting its total calibration.

It is well established that such friction increases as occur—due to use and surroundings—occur relatively early in the meter's life, the whole tendency being for the friction load to reach a substantially permanent condition after a comparatively brief period of service.

Instead of restoring the meter to its initial condition periodically by cleaning and the like, it becomes possible, where an adjustable compensation is available, to recompensate at the appropriate time for the permanent friction load, and thus restore the meter to a stable condition of equilibrium.

I have now laid before you the considerations which have led me to the conclusion that the two essentials meriting the most careful study in meter design are:

First, the expenditure of the maximum permissible amount of work in the foucault disk; and

Second, the provision of the best means for instantly compensating for friction variation at the point of use, without affecting total calibration.

A paper presented at the 18th Annual Convention of the American Institute of Electrical Engineers, Buffalo, August 21st, 1901. President Steinmetz in the Chair.

METERING OF ELECTRICAL ENERGY.

BY HARRY P. DAVIS.

The practice is now almost universal in the modern central station to use the integrating wattmeter for measuring and recording and from its reading to compute the charges to be made for supply to the consumers; consequently, it follows that the design of the wattmeter and the conditions under which it is operated, should be such as to omit of no question of its accuracy and permanency. These considerations, with the additional one of small cost of maintenance, are now very generally recognized as essential. As the meter accuracy thus bears so intimate a relation to the revenue return to the station, the meter becomes a factor in station engineering of prime importance, and as a result has received a vast amount of study, both at the hands of the user and of the manufacturer.

It is hoped that the following notes from the writer's experience in this field, although developing nothing particularly new, will outline some of the more important features which should be embodied in the ideal meter; and, for comparison with a commercial type, data and a brief description will be given of a test on a meter with which the writer has been most closely identified, and which is now largely used in practical operation.

While there are many characteristics which a meter must possess, only the more important will be pointed out as being essential to all types of meters. In considering specific features of design, however, it is the intention to cover only the so-called motor types of meters now so generally in use. Experience has

demonstrated that if a meter is electrically and mechanically well designed with reference to the law and conditions under which it is to operate, and proper care is given to its manufacture, it will be accurate and have the ability to remain in calibration. These features, namely, accuracy and permanency, are fundamental, and the meter which possesses them to a proper degree will surely possess all of the merits which are usually classed as fundamentals.

The meter which is theoretically correct will be influenced only by forces which are directly a product of the quantity which it is to measure. The revolving element will respond in exact proportion to the torque exerted upon it by this quantity and its changes in speed will in no way affect this proprotionality. The retarding force which will act in opposition will be independent of the quantity to be measured, and will increase in proper proportion with the speed of the revolving element.

A meter properly designed to embody these conditions, will measure accurately and be uninfluenced by changes in voltage or temperature, and when used in alternating current circuits will also record true energy under all conditions of inductive or non-inductive load, irrespective of variations in frequency or wave form. While seemingly simple, these conditions can never be completely attained, and many compromises must be made in the development of the commercial type of meter; obviously, however, the nearer they are approached the more accurate will be the meter, and the more extended will be its range of registration.

Commercially the range of accurate registration is most important; for the meter must account for all the conditions and variations of the load, without being delicate and expensive, qualifications requiring accuracy between wide limits, as it must start and record accurately on very light loads, and also be able to operate correctly under considerable overload. Further, the meter to be constant and unvarying in its operation, as it must be to be a practical and commercial success, the mechanical make-up must receive careful consideration, and it is most essential to have:

1st. Simplicity.

2d. Proper design of the rotating element.

3d. Proper design of the enclosing case.

The simple meter will have few parts, with every part easily accessible, and so related to others that the meter may be easily

taken down and put up without destroying or disturbing in any way the balance or adjustment of the separate parts. Compensating devices are necessary to extend the range and to straighten out the calibration curve. The fewer of these devices used, the better, however, and the amount of compensation needed should be slight, so that their influence may be small.

In the revolving element, we come to the consideration of what is by far the most important feature, in mechanical parts of the meter as a whole. The shaft must be short and stiff, and the entire revolving element must be simple and light in weight; this last consideration is most essential, affecting as it does in a vital manner the portability, life and ability of the meter to remain in calibration. When made sufficiently light all possibility is eliminated of injury to the jewel support, and the meter will respond instantly to sudden changes or fluctuation of load. Artificial, complicated and delicate devices for obviating injury to jewels or to act as substitutes for them, should be unnecessary if careful attention is given to these considerations in the design and proportions of the revolving element, for unless very well designed, such devices are liable in the long run to be troublesome.

No practicable form of bearing for a commercial meter in the writer's opinion, will have less friction or be more durable than a jewelled step bearing, provided the weight of the revolving element is not greater than a few grammes, with a speed of rotation as slow as is consistent with accurate registration on light loads. It is important also that meters used in alternating current circuits should be so proportioned that there will be no vibration effects, as they cause wear on the revolving element and its bearings, and in commercial use will attract the customer's attention and cause him dissatisfaction, owing to his belief that the noise evidences operation of the meter at times of no load.

Another and most potent factor having influence on the ability of a meter to remain in calibration is one entirely independent of the working parts of the meter proper, and yet requires as careful designing and proportioning in every way; and that is the enclosing case. This is a part of the meter, however, which, until recently, has been sadly neglected, and has resulted in being the cause of more trouble, in practical operation, than all other causes put together. A meter with a thin sheet metal

cover cannot be made tight, and soon becomes filled with dust and dirt, spiders, cockroaches, flies, etc. A meter is a precision instrument, and those with experience know the effect of a small amount of dirt on accuracy. The well designed case should be of light but substantial cast material, with the cover designed to make a tight joint with the base portion, and so arranged that it can be hermetically sealed, thereby effectually enclosing the interior of the meter. This requires the mechanism of the meter to be in a separate compartment from the terminals, but results to its advantage, as it enables connections to the circuit to be made, without any possibility of injury to the meter. A meter case made in this way will be dust, insect and moisture proof, and at the same time one which cannot be tampered with. These details, while purely mechanical, cannot be too highly emphasized, as they have so important a bearing on the permanency of the calibration curve. These are also points of design not always apparent and recognized for their full value on first examination and a test of a meter, but whose worth can only be established by the time test of actual operation.

The fact has been briefly noted of the qualifications and the wide limits which the calibration curve of the meter must possess to enable it to be elastic enough to meet the exacting requirements of the modern central station. As these requirements have been the keynote in meter development in the last five or six years, and as a proper appreciation of their value is most important in central station engineering, the matter is again referred to, and will be explained more in detail. The gradual increase in residential lighting to its present large proportion of every station's load presented new problems, many being of the greatest commercial importance. Residential lighting, especially in the districts far removed from the station, involves a high installation cost, with a corresponding low return per lamp installed. In these installations the maximum demand rarely exceeds one-third of the lamps connected, while one or two lamps may be burned for a considerable period. In central stations supplying such loads it is not an exaggeration to state that ten to fifteen per cent. of the 24 hour output is used in supplying individual installations, burning only one or two lamps for any great length of time, yet each having a total of 20 or 30 lamps installed. These are loads which, when measured by a meter

unsuited to their character, can be supplied indefinitely, with but slight return to the station, owing to the inability of the meter to register the small load with any degree of accuracy. This condition was becoming apparent as early as 1896, and much pressure was brought to bear on the manufacturers to produce a meter suitable to meet these requirements, namely, a meter with an unusually long range. Most of the difficulty was inability to start, and inaccurate registration on light loads, and the meters having no overload capacity it was necessary to install one having a normal capacity much greater than the average load to be measured. A meter was needed which would record accurately at least within two per cent. whether one lamp or the entire number installed, were burning, or, as previously stated, long range of registration in the meter was necessary. The fact was also apparent from the nature of the loads that if a meter could be designed which could stand a considerable overload for short periods, and register the same within fair accuracy, it would allow of a meter being installed of much smaller rated capacity than that of the installation, making it practicable to use a meter whose capacity would correspond to the average, instead of the maximum load of the installation. The use of a meter of small capacity would also have the advantage of giving a better registration for light loads, and proper selection of the size of meter to be installed would ensure accurate registration of the smallest load which would be placed on the circuit. Such were the conditions to be met, and meter design in the last few years has shown surprising development in this direction. In the central station, if careful attention is given to the fitting of the meter to the load to be measured with special reference to installations of this nature, where the necessity is so great for a meter which will start and record accurately at light load, and at the same time have ability to take care of an occasional overload, it is safe to say that a very considerable increase of revenue will result. This amount will be sufficient, in many instances where improper meters are in use, to pay in a very short time for the entire cost of the new meters and their installation, and to increase the revenue on the entire system by a very respectable percentage.

In a large central station, whose customers include many large apartment houses, and where particular attention has been paid to proper metering of this class of customers, it is customary, when a whole apartment house is to measure on one meter (the

entire installation will average 300 lamps, with about 12 lamps per apartment) to install a meter having a capacity of about 25 per cent. of the total number of lamps wired, and if each apartment is to be metered separately, a meter is used having 50 per cent. of the lights wired.

It is readily seen, that the use of meters of such small rated capacity, to that required for the total number of lamps installed, results in very accurate return of the light loads, which are usual for this class of installations. To perform such service, it requires great overload capability for the meter to be able to carry current now and then so greatly in excess of the normal, without a large drop and heating effect in the series coil. It is, therefore, important that this coil be of very low resistance, thus requiring it to be small, with few convolutions of wire. This will result in a small magnetizing force, and it is necessary in order to obtain the requisite turning moment on the revolving part, that the magnetic organization of the meter be nearly a closed iron circuit. It should be also noted in this connection that a series coil so designed will have a loss at full load, so slight as to be negligible, which is a valuable feature, as a large loss in the series coil of a meter causes a drop in voltage which will affect the regulation of the circuit, a loss, too, which occurs when the load on the system is the greatest. The use of a closed magnetic circuit has another important and useful bearing on the accuracy of the meter; as it makes it proof from the effects of external stray fields, concentrating as it does its own magnetizing field.

To show to what extent these conditions have been realized in commercial form, a single phase integrating wattmeter of Westinghouse manufacture was selected at random from stock for test. This wattmeter is of the induction type, in which the magnetic field produced by the windings induce secondary currents in an aluminium disk, causing it to rotate, in the same manner as the armature of an induction motor. The design of this meter has been carefully worked out, to give the best all around result in actual use. The coils are wound on iron cores, with very nearly closed magnetic circuits, thus greatly reducing the magnetizing forces necessary. This is particularly noticeable in the series coil which in an 80-ampere meter consists of but one turn of wire having a length of about three inches. Meters of other capacities have a proportionate number of turns. As so small a magnetizing force is required, it is evident that the

induction drop will be small, and as there are so few turns of wire, it can be made with a very low resistance, conditions which are very necessary to enable the meter to stand a heavy overload. The aluminium disk with its shaft is very light, weighing but 15 grammes.

Figure 1 is the calibration curve of the meter mentioned above. This shows that from 1/50 of full load, to 1½ times full load this meter is correct within 1½ per cent. and at twice full load it is still within 3 per cent. The small loss in series coil of this meter can best be appreciated when it is stated that during this test it carried without injury to itself for a considerable period a load of six times the rated full load capacity of the meter, representing a heating effect in the series circuit of 36 times that occurring at its normal rating.

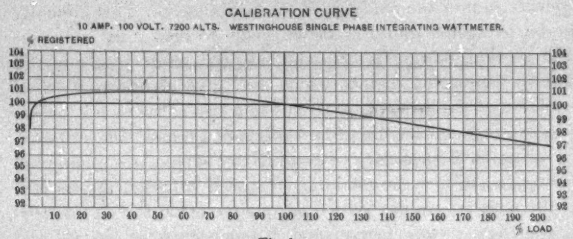


Fig. 1.

The writer would not consider this paper complete without a brief reference to the metering of polyphase currents, as this subject has now become a question of importance, owing to the rapid increase in the use of polyphase apparatus. The type of meter to use, the way to read the counters, the manner of connecting, and the limitations of the various connections, are not very generally understood, causing a great many difficulties and questions to be constantly arising. The energy delivered to a polyphase circuit can be measured by the use of one single-phase wattmeter, two single-phase wattmeters, or one polyphase wattmeter. A two-phase circuit may be treated as two similar single-phase circuits, and with a single-phase meter connected in one of them, its reading will indicate one-half the total energy supplied in the two circuits. It is obvious that to have the readings of any value for both circuits, it is absolutely necessary that the loads on the two phases be balanced; that is, of the same amount.

The use of a single-phase meter on a three-phase circuit is more complicated, as it is necessary to make special connections to obtain practicable readings, the connections being such as to make the meter show one-half of the total energy in the whole circuit. In order to have the reading, however, of total energy correct, the loads on the phases must be balanced.

With two single-phase meters, a two-phase circuit can be metered correctly whether balanced or unbalanced, the sum of the readings of the two meters in all cases giving the total energy in the circuit.

In a three-phase circuit the same is true, except that the meters will not give similar readings when the power factor of the circuit is less than unity, and with the power factor lower than 0.5 one of the meters will reverse and run backwards. It is necessary, therefore, in order to arrive at the total energy, to use the algebraic sum of the readings of the two meters. This difficulty can be overcome and the meter made to read equal amounts, with varying power factors and balanced loads, by the use of specially connected meters, which will give an apparent transformation of the three-phase circuit to a two-phase circuit before it is measured. The use of two single-phase meters is undesirable, owing to the necessity of reading two meters and adding the reading, to arrive at the total energy, and it is further aggravated by the negative values shown on one meter on three-phase circuits of low power factor, and if connected to obviate the trouble, the connections are difficult and complicated to be made.

The use of a polyphase meter overcomes all of these difficulties, as it can be connected either to a two or three-phase circuit, the connections being straightforward, and what is most important, its reading will be on a single dial, and will show the total energy irrespective of unbalancing of the loads or changes in power factor. It can be seen, therefore, that the whole question of using single-phase meters on polyphase circuits depends entirely on the balancing of the loads on the several phases, and the power factor of the circuit. In view of the uncertainty of the polyphase circuits in this respect, the exclusive use of polyphase meters is to be recommended, ensuring, as they do, accurate registration without regard to these irregularities.

A paper presented at the 18th Annual Convention of the American Institute of Electrical Engineers, Buffalo, August 22nd, 1901, Vice-President Dunn in the Chair.

THEORETICAL INVESTIGATION OF SOME OSCILLATIONS OF EXTREMELY HIGH POTENTIAL IN ALTERNATING HIGH POTENTIAL TRANSMISSIONS.

BY CHARLES PROTEUS STEINMETZ.

In the following I intend to give a section of a series of investigations extending over some years, on the effect of the exponential term in the general equation of alternating current:

If a sine wave of E. M. F. is impressed upon a general electric circuit containing resistance, inductance and capacity, the ultimate current is a sine wave also. In the moment of closing the circuit, however, or in any other way changing the circuit conditions, upon the sine wave of current an exponential term is superimposed, of the general character:

$$\Sigma A_1 \varepsilon^{-a_1 \varphi}$$

In a single circuit containing only resistance r and inductive reactance x, only one exponential term

$$A \varepsilon^{\frac{r}{x} \varphi}$$

appears. In a system of such circuits, directly or inductively connected with each other, a series of exponential terms appear,

the exponents of which are determined as the roots of an equation of nth degree.

If, however, a circuit contains resistance, inductance and capacity in series, two exponential terms

$$A_1 \varepsilon^{-a_1 \varphi} + A_2 \varepsilon^{-a_2 \varphi}$$

exist, where a_1 and a_2 are the roots of a quadratic equation, and either real or complex imaginary. In the latter case the exponential term resolves into the form:

$$\varepsilon^{-b \varphi} \left\{ B_1 \cos \varphi + B_2 \sin \varphi \right\}$$

an electric oscillation.

With several circuits of general character connected with each other directly or inductively, a series of exponential terms

$$\Sigma A_1 \varepsilon^{-a_1 \varphi}$$

appear, in which the exponents a_1 are the roots of an equation of $2n$th degree, which, when complex imaginary, give electrical oscillations, and the coefficients A_1 depend on the initial conditions of the circuits. Thus in such a system of circuits a number of superimposed oscillations may occur, of different frequencies and intensities.

Under certain conditions by such oscillations caused by a change of circuit conditions, destructive voltages may be produced.

The first time I noticed such a phenomenon was when making a series of observations on the effect of opening a high potential circuit with different types of circuit-breakers, on an artificial line consisting of inductive coils and about half a mile of high potential cable, with a spark gap between needle points connected across the system, to measure an instantaneous rise of voltage. If the needle points were brought so close together that a discharge occurred between them, invariably by the short-circuiting of the system across the needle points, an extremely high voltage was induced locally in the system, of a total striking distance reaching as high as 40″, that is between one-quarter and one-half

million volts, the normal voltage of the system being about 10,000.

The phenomenon is probably as follows: By striking between the needle points shunting the circuit, the system was short-circuited and thus instantly drained of its electro-static charge, so that the potential locally fell to practically nothing, while an excessive current passed between the needle points as arc. By the heating effect of this current the arc was instantly blown out explosively and thus the short-circuit current of the system ruptured. After the explosion the discharge between the needle points again short-circuited, interrupted itself, etc. The phenomenon is thus of similar character, but infinitely greater power, than the action of the Wehnelt interrupter, the latter depending upon the combined effect of self-induction and electrolytic polarization, while the phenomenon I observed occurred with self-induction and capacity.

Since that time in a number of instances of high potential circuits electrical discharges were noticed, of striking distances many times greater than that of the normal potential of the system, under circumstances where a short-circuiting arc interrupted itself in the open air. A short-circuit in a constrained space, as a buried cable, does not appear to give this effect.

From this and other reasons I was led to believe that a short-circuiting high potential arc in free space, in a system containing self-induction and capacity, is a self-interrupting phenomenon, and thus may cause very serious high potential effects. On any high potential limited power transformer this phenomenon can be observed by an entire change of the character of the arc between the high potential terminals when inserting capacity in shunt. From the flame of the steady and noiseless arc, which curves upward by its heat, the discharge changes to the glaring white, noisy, lightning-like, destructive high potential oscillation. This change can be shown photographically, the photographs giving in the former case, that is absence of capacity, the striations of the successive half waves of the impressed frequency, while in the latter case they show a system of successive electrostatic sparks not followed by the striated alternating arc flame. As shown by Mr. W. S. Andrews, a number of oscillating discharges may occur during the same half wave of alternating primary E. M. F.

The investigation of the character of electric oscillation pro-

duced by a short-circuiting and self-interrupting arc is the purpose of the following. The bearing of this phenomenon on high potential engineering is obvious.

If in a transmission line of impressed voltage $e = E \cos(\varphi - \omega)$ carrying current $I \cos(\varphi - \omega - \delta)$, the circuit is opened at the receiving end, there remain in circuit the line capacity in series with the resistance and inductance of step-up transformers, etc., and a part of the line.

Let, as an approximation, the line capacity be represented by a condenser shunted across the center of the transmission line, and let:

r = resistance,

L = inductance, thus: $x = 2\pi N L$ = reactance of circuit between constant potential generator bus-bars and line capacity (reduced to the line voltage).

Let:

C = capacity, thus: $k = \dfrac{1}{2\pi N C}$ = capacity reactance of line.

Let now the circuit be opened at the moment: $t = o$, or: $\varphi = o$, and at the moment when the circuit is opened:

e_0 = potential difference at line condenser,

i_0 = current in line.

If: i = instantaneous value of current at time t, it is: E. M. F. consumed by self-induction:

$$L\frac{di}{dt}$$

Potential difference at condenser terminals:

$$e_1 = \frac{1}{C}\int i\,dt$$

Thus the differential equation of the phenomenon is:

$$e - ir - L\frac{di}{dt} - \frac{1}{C}\int i\,dt = o$$

or, substituting:

$$\varphi = 2\pi N t,$$

$$e = E\cos(\varphi - \omega)$$

it is:

$$E\cos(\varphi-\omega)-ir-x\frac{di}{d\varphi}-k\int i\,d\varphi=0 \qquad (1)$$

and:

$$e_1=k\int i\,d\varphi=E\cos(\varphi-\omega)-ir-x\frac{di}{d\varphi} \qquad (2)$$

Equation (1) differentiated:

$$E\sin(\varphi-\omega)+k\,i+r\frac{di}{d\varphi}+x\frac{d^2i}{d\varphi^2}=0 \qquad (3)$$

This differential equation of second order is integrated by:

$$i=A\,\varepsilon^{-a\varphi}+B\cos(\varphi-\sigma) \qquad (4)$$

Substituting (4) in (3) gives:

$$(a^2x-ar+k)\,A\,\varepsilon^{-a\varphi}+\sin(\varphi+\sigma)\,[E\cos(\sigma-\omega)-rB]$$
$$+\cos(\varphi-\sigma)\,[E\sin(\sigma-\omega)-(x-k)\,B] \qquad (5)$$

Hence:

$$\left.\begin{aligned}&a^2x-ar+k=0\\&E\cos(\sigma-\omega)-rB=0\\&E\sin(\sigma-\omega)-(x-k)\,B=0\end{aligned}\right\} \qquad (6)$$

$$A=\text{indefinite},$$

and herefrom:

$$\tan(\sigma-\omega)=\frac{x-k}{r}$$

or, by denoting:

$$\tan\alpha=\frac{k-x}{r} \qquad (7)$$

where a is a constant of the circuit,

$$\sigma=\omega-\alpha \qquad (8)$$

Further:

$$B=\frac{E}{\sqrt{r^2+(x-k)^2}}$$

or, denoting:

$$z_0 = \sqrt{r^2 + (x - k)^2} \tag{9}$$

where z_0 is a constant of the circuit,

$$B = \frac{E}{z_0} \tag{10}$$

and α is, as root of a quadratic equation:

$$a = \frac{r \pm \sqrt{r^2 - 4\,x\,k}}{2\,x}$$

Since the inductive reactance x is of the same magnitude as the resistance r, the capacity reactance k, however, that is the reactance of the line as condenser, of higher magnitude than r or x, it is always:

$$4\,x\,k > r^2$$

that is, $\sqrt{r^2 - 4\,x\,k}$ is imaginary

Substituting:

$$q = \sqrt{4\,x\,k - r^2} \tag{11}$$

it is:

$$a = \frac{r \pm j\,q}{2\,x} \tag{12}$$

and the integral equation of the problem is:

$$i = A_1\,\varepsilon^{-\frac{r+jq}{2x}\varphi} + A_2\,\varepsilon^{-\frac{r-jq}{2x}\varphi} + \frac{E}{z_0}\cos(\varphi - \omega + \alpha) \tag{13}$$

that is:

$$i = \varepsilon^{-\frac{r}{2x}\varphi}\left\{A_1\,\varepsilon^{-j\frac{q}{2x}\varphi} + A_2\,\varepsilon^{+j\frac{q}{2x}\varphi}\right\} + \frac{E}{z_0}\cos(\varphi - \omega + \alpha)$$

Substituting the trigonometric expressions:

$$\left.\begin{aligned}\varepsilon^{+j\frac{q}{2x}\varphi} &= \cos\frac{q}{2\,x}\varphi + j\sin\frac{q}{2\,x}\varphi\\ \varepsilon^{-j\frac{q}{2x}\varphi} &= \cos\frac{q}{2\,x}\varphi - j\sin\frac{q}{2\,x}\varphi\end{aligned}\right\}$$

and rearranging, it is:

$$i = \varepsilon^{-\frac{r}{2x}} \left\{ (A_1 + A_2) \cos \frac{q}{2x} \varphi - j (A_1 - A_2) \sin \frac{q}{2x} \varphi \right\}$$

$$+ \frac{E}{z_0} \cos (\varphi - \omega + \alpha)$$

or, substituting:

$$\left.\begin{aligned} A_1 + A_2 &= C_1 \\ j (A_1 - A_2) &= C_2 \end{aligned}\right\}$$

it is:

$$i = \varepsilon^{-\frac{r}{2x}\varphi} \left(C_1 \cos \frac{q}{2x} \varphi - C_2 \sin \frac{q}{2x} \varphi \right) + \frac{E}{z_0} \cos(\varphi - \omega + \alpha) \quad (14)$$

where

C_1 and C_2 are determined by the circuit conditions at $t = 0$.

The potential difference at the condenser terminals is, by substituting (14) in (2):

$$e_1 = \varepsilon^{-\frac{r}{2x}\varphi} \left\{ C_1 \left(\frac{q}{2} \sin \frac{q}{2x} \varphi - \frac{r}{2} \cos \frac{q}{2x} \varphi \right) + C_2 \left(\frac{q}{2} \cos \frac{q}{2x} \varphi \right. \right.$$

$$\left. \left. + \frac{r}{2} \sin \frac{q}{2x} \varphi \right) \right\}$$

$$+ E \left\{ \cos (\varphi - \omega) - \frac{r}{z_0} \cos (\varphi - \omega + \alpha) + \frac{x}{z_0} \sin (\varphi - \omega + \alpha) \right\}$$

or, substituting:

$$\frac{r}{z_0} = \cos \alpha \quad \frac{x}{z_0} = \frac{k}{z_0} + \sin \alpha:$$

$$e_1 = \varepsilon^{-\frac{r}{2x}\varphi} \left\{ C_1 \left(\frac{q}{2} \sin \frac{q}{2x} \varphi - \frac{r}{2} \cos \frac{q}{2x} \varphi \right) \right.$$

$$\left. + C_2 \left(\frac{q}{2} \cos \frac{q}{2x} \varphi + \frac{r}{2} \sin \frac{q}{2x} \varphi \right) \right\}$$

$$+\frac{k}{z_0} E \sin(\varphi - \omega + \alpha) \quad (15)$$

At time: $t = 0$, or: $\varphi = 0$, it is, substituting in (14) and (15)

$$\left.\begin{aligned} i_0 &= C_1 + \frac{E}{z_0}\cos(\omega + \alpha) \\ e_0 &= -\frac{r}{2} C_1 + \frac{q}{2} C_2 - \frac{k}{z_0} E \sin(\omega + \alpha) \end{aligned}\right\}$$

and herefrom:

$$\left.\begin{aligned} C_1 &= i_0 - \frac{E}{z_0}\cos(\omega + \alpha) \\ C_2 &= \frac{2 e_0 + r i_0}{q} + \frac{k}{q z_0}[2 k \sin(\omega + \alpha) - r \cos(\omega + \alpha)] \end{aligned}\right\} \quad (16)$$

Substituting (16) in (14) and (15), gives:

Current:

$$i = \frac{E}{z_0}\cos(\varphi - \omega + \alpha) + \varepsilon^{-\frac{r}{2x}\varphi}\left\{\left[i_0 - \frac{E}{z_0}\cos(\omega + \alpha)\right]\cos\frac{q}{2x}\varphi - \left[\frac{2 e_0 + r i_0}{q} + \frac{E}{q z_0}\left(2 k \sin(\omega + \alpha) - r\cos(\omega + \alpha)\right)\right]\sin\frac{q}{2x}\varphi\right\} \quad (17)$$

Potential Difference at Condenser:

$$e_1 = \frac{k}{z_0} E \sin(\varphi - \omega + \alpha) + \varepsilon^{-\frac{r}{2x}\varphi}\left\{\left[e_0 + \frac{k}{z_0} E \sin(\omega - \alpha)\right]\cos\frac{q}{2x}\varphi + \left[\frac{2 r e_0 + (r^2 + q^2) i_0}{2 q} + \frac{E}{2 q z_0}(2 k r \sin(\omega - \alpha) - (r^2 + q^2)\cos(\omega - \alpha)\right]\sin\frac{q}{2x}\varphi\right\}$$

These equations (17) can be essentially simplified by neglecting terms of secondary magnitude.

k is in high potential transmission lines or cables always very large compared with r and x.

The full load resistance and reactance voltage may vary from less than five per cent. to about 20 per cent. of impressed voltage, the charging current of the line from five per cent. to about 20 per cent. of full load current, at normal voltage and frequency.

In this case, k is from 25 to more than 400 times as large as r or x, and r and x thus negligible compared with k.

It is then, in close approximation:

$$\left.\begin{aligned} z_0 &= k \\ q &= 2\sqrt{x\,k} \\ B &= \frac{E}{k} \\ \alpha &= \frac{\pi}{2} = q\ 0^0 \end{aligned}\right\} \tag{18}$$

Substituting these values in equations (17), gives:

Current:

$$i = -\frac{E}{k}\sin(\varphi - a) + \varepsilon^{-\frac{r}{2x}\varphi}\left\{\left[i_0 - \frac{E}{k}\sin\alpha\right]\cos\sqrt{\frac{k}{x}}\,\varphi - \left[\frac{2\,e_0 + r\,i_0}{2\sqrt{x\,k}} - \frac{E}{2\sqrt{x\,k}}(2\cos\omega + \frac{r}{k}\sin\omega)\right]\sin\sqrt{\frac{k}{x}}\,\varphi\right\} \tag{19}$$

Potential Difference at Condenser:

$$e_1 = E\cos(\varphi - \omega) + \varepsilon^{-\frac{r}{2x}\varphi}\left\{[e_0 - E\cos\omega]\cos\sqrt{\frac{k}{x}}\,\varphi + \left[\frac{2\,r\,e_0 + (r^2 + 4\,x\,k)\,i_0}{4\sqrt{x\,k}} - \frac{E}{4\sqrt{x\,k}}(2\,r\cos\omega + \frac{r^2 + 4\,x\,k}{k}\sin\varepsilon)\right]\sin\sqrt{\frac{k}{x}}\,\varphi\right\}$$

These equations consist of three terms:

$$\left.\begin{aligned} i &= i' + i'' + i''' \\ e_1 &= e_1' + e_1'' + e_1''' \end{aligned}\right\} \tag{20}$$

$$\left.\begin{aligned} i' &= -\frac{E}{k}\sin(\varphi-\omega) \\ e_1' &= E\cos(\varphi-\omega) \end{aligned}\right\} \tag{21}$$

$$\left.\begin{aligned} i'' &= -\frac{E}{k}\,\varepsilon^{-\frac{r}{2x}\varphi}\left\{\sin\omega\cos\sqrt{\frac{k}{x}}\,\varphi-\left[\sqrt{\frac{k}{x}}\cos\omega\right.\right. \\ &\qquad \left.\left.+\frac{r}{2\sqrt{xk}}\sin\omega\right]\sin\sqrt{\frac{k}{x}}\,\varphi\right\} \\ e_1'' &= -E\,\varepsilon^{-\frac{r}{2x}\varphi}\left\{\cos\omega\cos\sqrt{\frac{k}{x}}\,\varphi+\left[\frac{r}{2\sqrt{x\,k}}\cos\omega\right.\right. \\ &\qquad \left.\left.+\frac{r^2+4\,x\,k}{4\,k\,\sqrt{x\,k}}\sin\omega\right]\sin\sqrt{\frac{k}{x}}\,\varphi\right\} \end{aligned}\right\} \tag{22}$$

or, by dropping terms of secondary order:

$$\left.\begin{aligned} i'' &= \frac{E}{\sqrt{x\,k}}\,\varepsilon^{-\frac{r}{2x}\varphi}\cos\omega\sin\sqrt{\frac{k}{x}}\,\varphi \\ e_1'' &= -E\,\varepsilon^{-\frac{r}{2x}\varphi}\cos\omega\cos\sqrt{\frac{k}{x}}\,\varphi \end{aligned}\right\} \tag{23}$$

$$\left.\begin{aligned} i''' &= \varepsilon^{-\frac{r}{2x}\varphi}\left\{i_0\cos\sqrt{\frac{k}{x}}\,\varphi-\frac{2\,e_0+r\,i_0}{2\sqrt{x\,k}}\sin\sqrt{\frac{k}{x}}\,\varphi\right\} \\ e_1''' &= \varepsilon^{-\frac{r}{2x}\varphi}\left\{e_0\cos\sqrt{\frac{k}{x}}\,\varphi+\frac{2re_0+(r^2+4xk)i_0}{4\sqrt{x\,k}}\sin\sqrt{\frac{k}{x}}\,\varphi\right\} \end{aligned}\right\} \tag{24}$$

or, by dropping terms of secondary order:

$$\left.\begin{aligned} i''' &= \varepsilon^{-\frac{r}{2x}\varphi}\left\{i_0\cos\sqrt{\frac{k}{x}}\,\varphi-\frac{e_0}{\sqrt{xk}}\sin\sqrt{\frac{k}{x}}\,\varphi\right\} \\ e_1''' &= \varepsilon^{-\frac{r}{2x}\varphi}\left\{e_0\cos\sqrt{\frac{k}{x}}\,\varphi+i_0\sqrt{x\,k}\,\sin\sqrt{\frac{k}{x}}\right\} \end{aligned}\right\} \tag{25}$$

Thus:

Total Current, approx:

$$i = -\frac{E}{k}\sin(\varphi-\omega)+\varepsilon^{-\frac{r}{2x}\varphi}\left\{ i_0\cos\sqrt{\frac{k}{x}}\,\varphi - \frac{e_0 - E\cos\omega}{\sqrt{x\,k}}\sin\sqrt{\frac{k}{x}}\,\varphi\right\}$$

Difference of Potential at Condenser:

$$e_1 = E\cos(\varphi-\omega)+\varepsilon^{-\frac{r}{2x}\varphi}\left\{(e_0-E\cos\omega)\cos\sqrt{\frac{k}{x}}\,\varphi + i_0\sqrt{x\,k}\sin\sqrt{\frac{k}{x}}\,\varphi\right\} \qquad (26)$$

Of the three terms: i', e_1'; i'', e_1''; i''', e_1''', the first obviously represents the stationary condition of charging current and condenser potential, since the two other terms disappear for $t=\infty$.

The second term, i'', e_1'', represents that component of oscillation which depends upon the phase of impressed E. M. F., or the point of the impressed E. M. F. wave, at which the oscillation begins, while the third term, i''', e_1''', represents the component of oscillation which depends upon the instantaneous values of current resp. E. M. F. at the moment at which the oscillation begins. $\varepsilon^{-(r/2x)\varphi}$ is the decrement of the oscillation.

The frequency of oscillation is:

$$N_0 = \sqrt{\frac{k}{x}}\,N$$

where N is the impressed frequency. That is, the frequency of oscillation equals the impressed frequency times the square root of the ratio of capacity reactance and inductive reactance of the circuit, or is the impressed frequency divided by the square root of inductance voltage and capacity current, as fraction of impressed voltage and full load current.

Since:

$$k = \frac{1}{2\pi N C},\quad x = 2\pi N L,$$

the frequency of oscillation is:

$$N_0 = \frac{1}{2\pi\sqrt{CL}}$$

that is, independent of the frequency of the impressed E. M. F.

Substituting:

$$\varphi = 2\pi N t, k = \frac{1}{2\pi N C}, \omega = 2\pi N L$$

in equations (23), (25), and (26), it is:

$$\left.\begin{aligned} i'' &= \sqrt{\frac{C}{L}}\, E\, \varepsilon^{-(r/2L)t} \cos\omega \sin\frac{t}{\sqrt{CL}} \\ e_1'' &= -E\,\varepsilon^{-(r/2L)t} \cos\omega \cos\frac{t}{\sqrt{CL}} \end{aligned}\right\} \quad (27)$$

$$\left.\begin{aligned} i''' &= \varepsilon^{-(r/2L)t}\left\{ i_0 \cos\frac{t}{\sqrt{CL}} - e_0\sqrt{\frac{C}{L}}\sin\frac{t}{\sqrt{CL}}\right\} \\ e_1''' &= \varepsilon^{-(r/2L)t}\left\{ e_0 \cos\frac{t}{\sqrt{CL}} + i_0\sqrt{\frac{L}{C}}\sin\frac{t}{\sqrt{CL}}\right\} \end{aligned}\right\} \quad (28)$$

$$\left.\begin{aligned} i &= -2\pi N C E \sin(\varphi-\omega) + \varepsilon^{-(r/2L)t}\left\{ i_0\cos\frac{t}{\sqrt{CL}}\right. \\ &\qquad \left. -(e_0 - E\cos\omega)\sqrt{\frac{C}{L}}\sin\frac{t}{\sqrt{CL}}\right\} \\ e_1 &= E\cos(\varphi-\omega) + \varepsilon^{-(r/2L)t}\left\{ (e_0 - E\cos\omega)\cos\frac{t}{\sqrt{CL}}\right. \\ &\qquad \left. + i_0\sqrt{\frac{L}{C}}\sin\frac{t}{\sqrt{CL}}\right\} \end{aligned}\right\} \quad (29)$$

The oscillating terms of these equations is independent of the impressed frequency. That is, the oscillating current and potential difference, caused by a change of circuit conditions (as starting, change of load or opening circuit), are independent of the impressed frequency, and thus also of the wave shape of the im-

pressed E. M. F., or its higher harmonics (except as regards to terms of secondary order).

The first component of oscillation, equations (22) and (23), besides upon the line constants and upon the impressed E. M. F., depends only upon the phase, or the point of the impressed E. M. F. wave, at which the oscillation starts, but does not depend upon the previous condition of the circuit.

Therefore this component of oscillation is the same as the oscillation produced in starting the transmission line, that is, connecting it, unexcited, to the generator terminals.

There exists no point of the impressed E. M. F. wave, where no oscillation occurs (while, when starting a circuit containing resistance and induction only, at the point of the impressed E. M. F. wave, where the final current passes zero, the stationary condition is instantly reached).

With capacity in circuit, any change of circuit conditions involves an electric oscillation.

The maximum intensity of starting oscillation occurs near the value: $a=o$, and is:

$$\left.\begin{aligned} i'' &= \frac{E}{\sqrt{x\,k}}\,\varepsilon^{-(r/2x)\varphi}\sin\sqrt{\frac{k}{x}}\,\varphi \\ e_1'' &= -E\,\varepsilon^{-(r/2x)\varphi}\cos\sqrt{\frac{k}{x}}\,\varphi \end{aligned}\right\} \quad (30)$$

Since:

$$i' = -\frac{E}{k}\sin(\varphi - \omega)$$

is the stationary value of charging current, it follows:

The maximum intensity which the oscillating current produced in starting a transmission line may reach, is $\sqrt{\frac{k}{x}}$ times the stationary charging current, or the initial rush of current bears to the stationary value the same ratio as the frequency of oscillation to the impressed frequency.

The maximum oscillating E. M. F. induced in starting a transmission line is of the same value as the impressed E. M. F. Thus the maximum value of potential difference occurring in a transmission line at starting is less than twice the impressed E. M. F. and no excessive voltages can be induced in starting a circuit.

The minimum value of starting oscillation occurs near: $\omega = 90°$, and is, from equations (22):

$$\left.\begin{aligned} i'' &= -\frac{E}{k}\,\varepsilon^{-(r/2x)\varphi}\cos\sqrt{\frac{k}{x}}\,\varphi \\ e_1'' &= \sqrt{\frac{x}{k}}\,E\,\varepsilon^{-(r/2x)\varphi}\sin\sqrt{\frac{k}{x}}\,\varphi \end{aligned}\right\} \quad (31)$$

That is, the oscillating current is of the same intensity as the charging current, and the maximum rush of current thus is less than twice the stationary value. The potential difference in the circuit rises only little above the impressed E. M. F.

The second component of oscillation, equations (24.) (25.), does not depend upon the point of the impressed E. M. F. wave, at which the oscillation starts, ω, nor upon the impressed E. M. F. as a whole, E, but, besides upon the constants of the circuit, it only depends upon the instantaneous values of current and of potential difference in the circuit at the moment where the oscillation starts, i_0 and e_0.

Thus, if: $i_0 = 0$, $e_0 = 0$, or in starting a transmission line, unexcited, by connecting it to the impressed E. M. F., this term disappears.

It is this component which may cause excessive potential differences.

Two cases shall more fully be discussed.

(*a*.) Opening the circuit of a transmission line under load.

(*b*.) Rupturing a short-circuit on the transmission line.

A.

If i_0 is the instantaneous value of full load current, e_0 the instantaneous value of difference of potential at the condenser, it is:

ri small compared with e_0:

$\sqrt{x\,k}\,i_0$ of the same magnitude as e_0.

Denoting:

$$\tan\delta = \frac{e_0}{i_0\sqrt{x\,k}}$$

it is, substituting in equations (24):

$$\left.\begin{aligned} i''' &= \sqrt{i_0^2 + \frac{e_0^2}{x\,k}}\;\varepsilon^{-(r/2x)\varphi}\cos\left(\sqrt{\frac{k}{x}}\,\varphi + \delta\right) \\ e_1''' &= \sqrt{e_0^2 + i_0^2\,x\,k}\;\varepsilon^{-(r/2x)\phi}\sin\left(\sqrt{\frac{k}{x}}\,\varphi + \delta\right) \end{aligned}\right\} \quad (32)$$

That is, the amplitude of oscillation is

$$\sqrt{i_0^2 + \frac{e_0^2}{x\,k}}$$

for the current, $\sqrt{e_0^2 + i_0^2\,x\,k}$ for the E. M. F. Thus the induced E. M. F. can be larger than the impressed E. M. F., but is, as a rule, still of the same magnitude, except if k is very large.

In the expression of the total current resp. potential difference at condenser, in equations (26.), $(e_0 - E\cos\omega)$ is the difference between the potential difference at the condenser and the impressed E. M. F., at the instant of starting of the oscillation, or the voltage consumed by the line impedance, and this is small, if the current is not excessive. Thus, neglecting the terms with $(e_0 - E\cos\omega)$, equations (26.) assume the form:

$$\left.\begin{aligned} i &= -\frac{E}{k}\sin(\varphi - \omega) + i_0\,\varepsilon^{-(r/2x)\phi}\cos\sqrt{\frac{k}{x}}\,\varphi \\ e_1 &= E\cos(\varphi - \omega) + i_0\sqrt{x\,k}\;\varepsilon^{-(r/2x)\phi}\sin\sqrt{\frac{k}{x}}\,\varphi \end{aligned}\right\} \quad (33)$$

That is, the oscillation of current is of the amplitude of full load current, and the oscillation of condenser potential is of the amplitude $i_0\sqrt{x\,k}$.

$x\,k$ is the ratio of inductance voltage to condenser current, in fractions of full load voltage and current. It is also:

$$i_0\sqrt{x\,k} = i_0\sqrt{\frac{L}{C}}$$

Thus in circuits of very high inductance L and relatively low capacity C, $i_0\sqrt{x\,k}$ may be much higher than the impressed E. M. F., and a serious rise of potential occur in opening the circuit under load, while in low inductance cables of high capacity,

$i_0\sqrt{x\,k}$ is moderate. That is, the inductance, by tending to maintain the current, induces a rise of potential, while capacity exerts a cushioning effect. Low inductance and high capacity thus are of advantage when breaking full load current in a circuit.

B.

If a transmission line containing resistance, inductance and capacity is short-circuited, and the short-circuit suddenly opened at time $t = 0$, it is, for: $t < 0$:

$$\left.\begin{aligned} e_0 &= 0 \\ i &= \frac{E}{z}\cos(\varphi - \omega - j) \\ \text{where:}\qquad z &= \sqrt{r^2 + x^2} \\ \tan j &= \frac{x}{r} \end{aligned}\right\} \quad (34)$$

thus, at time: $t = 0$

$$i_0 = \frac{E}{z}\cos(\omega + j) \qquad (35)$$

Substituting these values of e_0 and i_0 in equations (24), gives:

$$i''' = \frac{E}{z}\cos(\omega+j\ \varepsilon^{-(r/2x)\phi}\left\{\cos\sqrt{\frac{k}{x}}\varphi - \frac{r}{2\sqrt{x\,k}}\sin\sqrt{\frac{k}{x}}\varphi\right\}$$

$$e_1''' = \frac{E}{z}\cos(\omega + j)\ \varepsilon^{-(r/2x)\phi}\frac{r^2 + 4\,x\,k}{4\sqrt{x\,k}}\sin\sqrt{\frac{k}{x}}\varphi$$

or, neglecting terms of secondary magnitude:

$$\left.\begin{aligned} i''' &= \frac{E}{z}\ \varepsilon^{-(r/2x)\phi}\cos(\omega + j)\cos\sqrt{\frac{k}{x}}\varphi \\ e_1''' &= \frac{E\sqrt{x\,k}}{z}\ \varepsilon^{-(r/2x)\phi}\cos(\omega + j)\sin\sqrt{\frac{k}{x}}\varphi \end{aligned}\right\} \quad (36)$$

That is, i''' is of the magnitude of short-circuit current, and e_1''' of higher magnitude than the impressed E. M. F., since z is small compared with $\sqrt{x\,k}$.

The total values of current and condenser potential difference are, from equation (26):

$$\left.\begin{aligned} i &= -\frac{E}{k}\sin(\varphi-\omega)+E\,\varepsilon^{-(r/2x)\varphi}\left\{\left[\frac{\cos(\omega+j)}{z}\right.\right.\\ &\quad\left.\left.-\frac{\sin\omega}{k}\right]\cos\sqrt{\frac{k}{x}}\,\varphi+\frac{\cos\omega}{\sqrt{xk}}\sin\sqrt{\frac{k}{x}}\,\varphi\right\}\\ e_1''' &= E\cos(\varphi-\omega)-E\,\varepsilon^{-(r/2x)\varphi}\left\{\cos\omega\cos\sqrt{\frac{k}{x}}\,\varphi\right.\\ &\quad\left.-\frac{\sqrt{xk}\cos(\omega+j)}{z}\sin\sqrt{\frac{k}{x}}\,\varphi\right\} \end{aligned}\right\}\quad(37)$$

or approximately, since all terms are negligible compared with i''' and e_1''':

$$\left.\begin{aligned} i &= \frac{E}{z}\,\varepsilon^{-(r/2x)\varphi}\cos(\omega+j)\cos\sqrt{\frac{k}{x}}\,\varphi\\ e_1 &= \frac{E\sqrt{xk}}{z}\,\varepsilon^{-(r/2x)\varphi}\cos(\omega+j)\sin\sqrt{\frac{k}{x}}\,\varphi \end{aligned}\right\}\quad(38)$$

These values are a maximum, if the circuit is opened at the moment: $a=-j$, that is at the maximum value of the short-circuit current, and are then:

$$\left.\begin{aligned} i &= \frac{E}{z}\,\varepsilon^{-(r/2x)\varphi}\cos\sqrt{\frac{k}{x}}\,\varphi\\ e_1 &= \frac{\sqrt{xk}}{z}\,E\,\varepsilon^{-(r/2x)\varphi}\sin\sqrt{\frac{k}{x}}\,\varphi \end{aligned}\right\}\quad(39)$$

The amplitude of oscillation of condenser potential difference is:

$$\frac{\sqrt{xk}}{z}E$$

or, neglecting the line resistance, as rough approximation: $x=z$:

$$\sqrt{\frac{k}{x}}\,E$$

That is, the potential difference at the condenser is increased above the impressed E. M. F. in the proportion of the square root of the ratio of capacity reactance to inductive reactance, or inverse proportional to the square root of inductance voltage times capacity current, as fraction of impressed voltage and full load current.

Thus in this case, the rise of voltage is excessive.

The minimum intensity of the oscillation due to rupturing short-circuit occurs if the circuit is broken at the moment: $a = 90° - j$, that is, at the zero value of short-circuit current. It is then:

$$\left.\begin{aligned} i &= \frac{E}{\varphi}\cos(\varphi + j) - E\,\varepsilon^{-(r/2x)\varphi}\left\{\frac{\cos j}{\varphi}\cos\sqrt{\frac{\varphi}{x}}\,\varphi\right. \\ &\qquad \left. - \frac{\sin j}{\sqrt{x\varphi}}\sin\sqrt{\frac{\varphi}{x}}\,\varphi\right\} \\ e_1 &= E\sin(\varphi + j) - E\,\varepsilon^{-(r/2x)\varphi}\sin j\cos\sqrt{\frac{k}{x}}\,\varphi \end{aligned}\right\} \quad (40)$$

That is, the potential difference at the condenser is less than twice the impressed E. M. F., thus moderate.

Hence, a short-circuit can be opened safely only at or near the zero value of the short-circuit current.

The phenomenon ceases to be oscillating, and becomes an ordinary logarithmic discharge, if $\sqrt{r_2 - 4\,x\,k}$ is real, or:

$$r < 2\sqrt{x\,\varphi}.$$

Some instances may illustrate the phenomena discussed in the preceding:

I.

Let in a transmission line carrying 100 amperes at full load, under an impressed E. M. F. of 20,000 volts, the resistance drop equal 8%, the inductance voltage 15% of the impressed voltage, and the charging current 8% of full load current. Assuming as resistance drop in the step-up transformers, 1%, and as reactance voltage 2½%.

The resistance drop between constant potential generator terminals and middle of transmission line is then 5%, or $r = 10$ ohms. The inductance voltage is 10%, or $x = 20$ ohms. The

charging current of the line is 8 amperes, thus the capacity reactance $k = 2500$ ohms.

It is then, assuming a sine wave of impressed E. M. F.:

$$E = 20000\ \sqrt{2} = 28280 \text{ volts.}$$

$$i' = -\ 11.3 \sin (\varphi - \omega)$$
$$e_1' = 28280 \cos (\varphi - \omega)$$

$$i'' = -\ 11.3\ \varepsilon^{-.25\,\phi} [\sin \omega \cos 11.2\ \varphi - 11.2 \cos \omega \sin 11.2\ \varphi]$$
$$e_1'' = -\ 28280\ \varepsilon^{-.25\,\phi} [\cos \omega \cos 11.2\ \varphi - .089 \sin \omega \sin 11.2\ \varphi]$$

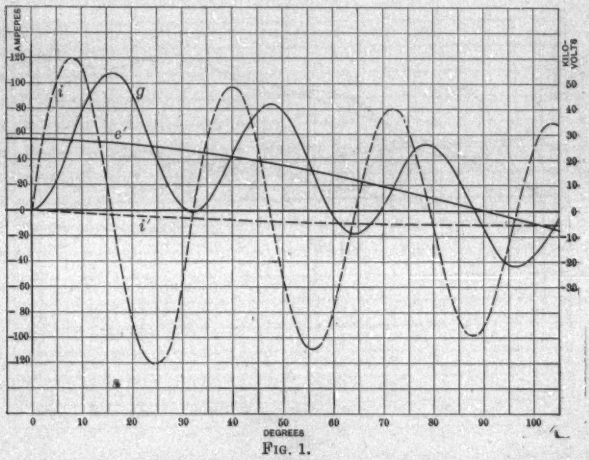


FIG. 1.

Thus, the oscillation produced in starting the transmission line is:

$$i = -\ 11.3 [\sin (\varphi - \omega) + \varepsilon^{-.25\,\phi} (\sin \omega \cos 11.2\ \varphi - 11.2 \cos \omega \sin 11.2\ \varphi)]$$
$$e_1 = 28280\ [\cos (\varphi - \omega) - \varepsilon^{-.25\,\phi} (\cos \omega \cos 11.2\ \varphi - .089 \sin \omega \sin 11.2\ \varphi)]$$

Hence the maximum value, for: $\omega = 0$:

$$i = -\ 11.3 (\sin \varphi - 11.2\ \varepsilon^{-.25\,\phi} \sin 11.2\ \varphi)$$
$$e_1 = 28280 (\cos \varphi - \varepsilon^{-.25\,\phi} \cos 11.2\ \varphi)$$

and the minimum value, for: $\omega = 90°$:

$$i = 11.3\ (\cos\varphi - \varepsilon^{-.25\,\varphi} \cos 11.2\ \varphi)$$
$$e_1 = 28280\ (\sin\varphi + .089\ \varepsilon^{-.25\,\varphi} \sin 11.2\ \varphi)$$

These values are plotted in Figs. 1 and 2, with the current i in dotted and the potential difference e_1 in drawn line. The stationary values are plotted also, in thin lines.

(*a*.) Opening the circuit under full load, it is:

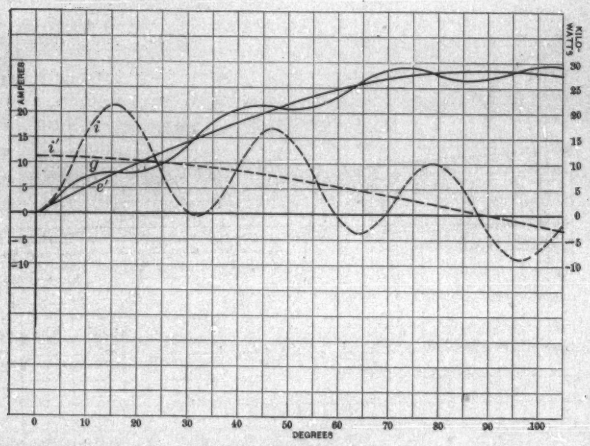


FIG. 2.

$$i = -11.3 \sin(\varphi - \omega) + \varepsilon^{-.25\,\varphi}(i_o - 11.3 \sin\omega) \cos 11.2\ \varphi$$
$$e_1 - 28280 \cos(\varphi - \omega) + 224\ i_o\ \varepsilon^{-.25\,\varphi} \sin 11.2\ \varphi$$

These values are maximum for $\omega = 0$ and non-inductive circuit, or: $i_o = 141.4$, and are:

$$i = -11.3 \sin\varphi + 141.4\ \varepsilon^{-.25\,\varphi} \cos 11.2\ \varphi$$
$$e_1 = 28280 \cos\varphi + 31600\ \varepsilon^{-.25\,\varphi} \sin 11.2\ \varphi$$

These values are plotted, in Fig. 3, in the same manner as Figs. 1 and 2:

(*b.*) Rupturing the line under short-circuit, it is:

$$z = 22.4$$
$$i_0 = 1265 \cos (\omega + j$$

thus:

$$i = -11.3 \sin (\varphi - \omega) + 1265\, \varepsilon^{-.25\,\varphi} \{[\cos (\omega + j) - .0089 \sin \omega] \cos 11.2\, \varphi + .1 \cos \omega \sin 11.2\, \varphi\}$$
$$e_1 = 28280 \{\cos (\varphi + \omega) - \varepsilon^{-.25\,\varphi} [\cos \omega \cos 11.2\, \varphi - 10 \cos (\omega + j) \sin 11.2\varphi]\}$$

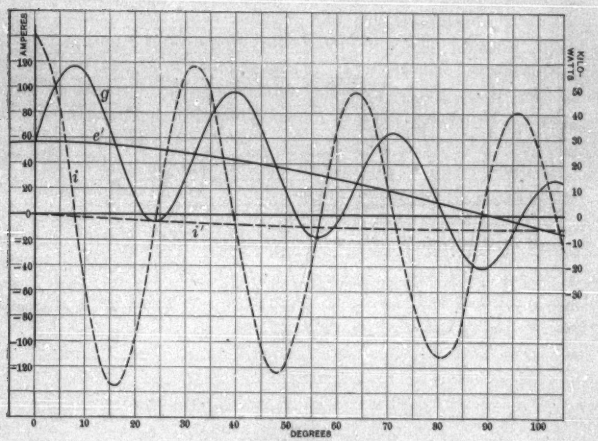


FIG. 3.

These values are a maximum for: $\omega = -j = -63°$:

$$i = -11.3 \sin (\varphi + 63°) + 1260\, \varepsilon^{-.25\,\varphi} (\cos 11.2\, \varphi + .044 \sin 11.2\, \varphi)$$
$$e_1 = 28280 \cos (\varphi + 63°) - 282{,}800\, \varepsilon^{-.25\,\varphi} (.044 \cos 11.2\, \varphi - \sin 11.2\, \varphi)$$

That is, the potential difference rises about tenfold, to 282,800 volts.

These values are plotted in Fig. 4.

II.

In the instance referred to in the beginning of the paper, where I first observed the destructive voltage induced by a short-circuiting arc, on an experimental 10,000-volt 40-cycle line, during the occurrence of the phenomenon the generator voltage probably fell to about 5,000, due to limited capacity. The resistance of the system was very low, about $r = 1$ ohm, while the inductive reactance may be estimated as $x = 10$ ohms, and the capacity reactance as $k = 20{,}000$ ohms.

It is thus: $\tan j = 10$, or approximately: $j = 90°$.

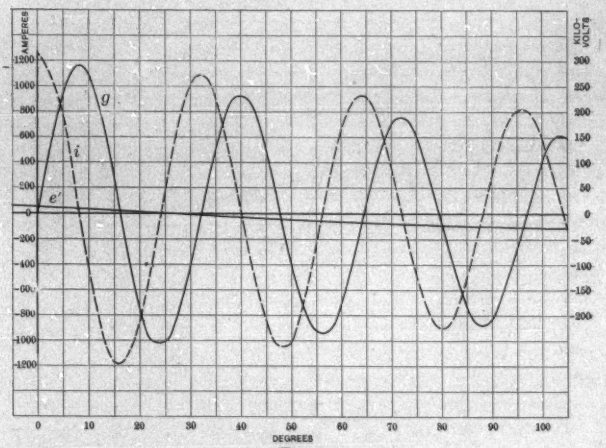


FIG. 4.

Herefrom it follows:

$$i = 707\ \varepsilon^{-.05\,\varphi} \cos 44.7\ \varphi$$
$$e_1 = 316000\ \varepsilon^{-.05\,\varphi} \sin 44.7\ \varphi$$

That is, the oscillation has a frequency of about 1,800 cycles, and a maximum E. M. F. of nearly one-third million volts, which fully accounts for its disruptive effects.

As conclusion, it follows herefrom:

1. The most important source of destructive high voltage phenomena in high potential circuits containing inductance and capacity are not resonance phenomena with the wave of im-

pressed E. M. F. or its higher harmonics, in case of the deviation of the wave from sine shape, but are the electric oscillations produced by a change of circuit conditions, as starting, opening circuit, etc.

2. These phenomena are essentially independent of frequency and wave shape of impressed E. M. F., but depend upon the conditions under which the circuit is changed, as the manner of change and the point of the impressed E. M. F. and current wave at which the change occurs.

3. The electric oscillations occuring in connecting a transmission line to the generator are not of dangerous potential, but the oscillations produced by opening the transmission circuit under load may reach destructive voltages, and the oscillations caused by interrupting a short-circuit are liable to reach voltages far beyond the strength of any insulation. Thus special precautions should be taken in opening a high potential circuit under load. But the most dangerous phenomenon is a low resistance short-circuit in open space.

4. The voltages produced by the oscillations in open-circuiting a transmission line under load or under short-circuit are moderate, if the opening of the circuit occurs at a certain point of the E. M. F. wave. This point approximately coincides with the moment of zero current.

March, 1901.
Schenectady, N. Y.

A paper presented at the 157th Meeting of the American Institute of Electrical Engineers, New York, October 25th, 1901, President Steinmetz in the Chair.

ANGULAR VARIATION IN STEAM ENGINES.

BY P. O. KEILHOLTZ.

Introduction.

The following investigation was undertaken with the view of determining the angular variation of a 29 x 60 x 56 cross compound vertical engine built by McIntosh, Seymour & Co., Auburn, N. Y., and direct connected to a M. P. 22–2000–100, 500-volt railway generator built by the General Electric Co. It is understood of course that the variation of angular velocity does not interfere with the successful parallel operation of direct current generators, but as the variation is caused by the engine, it is immaterial to this investigation as to the type of apparatus driven.

The paper is divided into two parts, theoretical and experimental. In the former part I have consulted and appropriated from papers written by the late Prof. Fleeming Jenkin, *Transactions of the Royal Society of Edinburgh*, vol. xxviii., 1879; and J. H. Macalpine, Esq., *Journal of the American Society of Naval Engineers*, vol. ix., No. 3. There is, however, sufficient originality to warrant giving the theoretical part in full.

I have been informed that the angular variation of an engine has been measured by driving a small direct current generator from the rim of the fly-wheel of the engine and balancing the E. M. F. of the generator by a storage battery. A voltmeter with a minimum amount of damping was then connected across the terminals of the generator and the deflection read. This would show the total velocity variation of the fly-wheel from which the

space variation with a wheel revolving at constant speed could be calculated.

As before stated, it was intended to measure the angular variation of the 29x60x56 vertical cross-compound engine, but owing to the fact that the engine cannot be spared at the present time, the measurement was made upon a 24 x 45 x 42 horizontal tandem compound engine built by McIntosh, Seymour & Co., and direct connected to a M. P. 10–800–100 500-volt railway generator built by the General Electric Company. The peripheral speed of the fly-wheel rim of the vertical is 90 feet per second, and the tandem 83¾ feet per second. The difficulties of measurement would be practically the same, although the space variation of the vertical would be less than the tandem. The curves of turning moments of both engines are given, together with curves of velocity and space variation, and it is my intention to measure the space variation of the vertical engine when the engine can be spared for the purpose.

The data for the curves of steam and inertia turning moments for the vertical engine are given in Tables X and XI. Table X is formed from equations 4 and '25, the substituted values giving

$$\frac{M}{g}\,\omega^2\,r^2\,[+\,.05\sin\theta - .5\sin 2\,\theta - .15\sin 3\,\theta - .01\sin 4\,\theta]$$

and

$$\frac{R}{g}\,\omega^2\,r\,[-.0025\sin\theta - .296\sin 2\,\theta - .0975\sin 3\,\theta - .0047\sin 4\,\theta]$$

The running parts were carefully weighed, and are as follows:

Low pressure piston, rod and nut weighed	3320	lbs.
High " " " " " "	1988.5	"
Connecting rod complete, high and low, each	2944	"
Crosshead " " " " "	1203.5	"
High pressure crank leads 90°		

The moment of inertia of the connecting rod about its center of gravity was obtained experimentally. The rod was swung about an axis which passed through the center of the wrist pin. A semi-circular wooden block to which was fastened a metal plate was inserted in place of the wrist pin, the whole resting

upon the edge of a 4 x 4 angle. The moment of friction could be neglected as the amplitude of the vibration was less than one inch. In this way its length as an equivalent simple pendulum was obtained. The distance of the center of gravity from the axis of suspension was found by balancing the rod upon the edge of a 4x4 angle. With these measured quantities the moment of inertia of the rod about its center of gravity was found, and reduced to the terms of the equation,

$$I_c = 3.4\, R\, r^2$$

where $K = 0.53$; and $a = 0.2$. (The vertical is 5 cranks and the tandem 6 cranks).

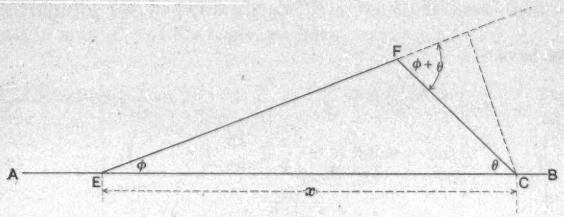


FIG. 1.

Length of connecting rod $E\,F$, l.
Length of crank radius $C\,F$, r.

$$\text{Call } \frac{r}{l} = a$$

Let a force P act at E in the direction $A\,B$ to find its turning moment about C.

Take origin of co-ordinates at C and call the distance $E\,C$, x, then the geometry of Figure 1 gives:

$$x = l \cos \varphi + r \cos \theta, \text{ and } \sin \varphi = a \sin \theta.$$

Substituting

$$x = \frac{r}{a}\,(1 - a^2 \sin^2 \theta)^{\frac{1}{2}} + r \cos \theta$$

Expanding the radical and substituting the values of the powers of sin θ in terms of cos 2 θ, cos 4 θ and cos 6 θ, we have:

$$x = \frac{r}{a}\left|\begin{array}{l} 1 - \frac{a^2}{4} - \frac{3\,a^4}{64} - \frac{10\,a^6}{256} - \ldots. \\ + \cos 2\theta\left(\frac{a^2}{4} + \frac{a^4}{16} + \frac{15\,a^6}{256} + \ldots\right) \\ - \cos 4\theta\left(\frac{a^4}{64} + \frac{6\,a^6}{256} + \ldots.\right) \\ + \cos 6\theta\left(\frac{a^6}{256} + \ldots.\right) \end{array}\right| + r\cos\theta \qquad (1)$$

Differentiating with respect to time and considering the angular velocity of the crank constant, and equal to

$$\frac{d\theta}{dt} = \omega$$

we have:

$$\frac{d^2x}{dt^2} = \omega^2 r\left|\begin{array}{l} -\cos\theta \\ -\cos 2\theta\,(a + \frac{a^3}{4} + \frac{15}{64}\,a^5 + \ldots.) \\ + \cos 4\theta\left(\frac{a^3}{4} + \frac{3}{8}\,a^5 + \ldots.\right) \end{array}\right| \qquad (2)$$

The resolved part of the force P along the connecting rod is

$$\frac{P}{\cos\varphi}$$

and its turning moment about the crank shaft is

$$-\frac{P}{\cos\varphi}\,r\sin(\theta + \varphi)$$

The development of

$$\frac{\sin(\theta + \varphi)}{\cos\varphi}$$

in terms of $\sin\theta$ and multiples of that angle is as follows:

$$\frac{\sin(\theta + \varphi)}{\cos\varphi} = \frac{\sin\theta\cos\varphi + \cos\theta\sin\varphi}{\cos\varphi}$$

$$= \sin\theta + \frac{a\cos\theta\sin\theta}{\sqrt{(1 - a^2\sin^2\theta)}}$$

$$= \sin\theta + \frac{a}{2}\sin 2\theta\,(1 - a\,\sin^2\theta)^{-\frac{1}{2}}$$

Expanding the quantity in parenthesis we have

$$\frac{\sin(\theta+\varphi)}{\cos\varphi} = \sin\theta + \frac{a}{2}\sin 2\theta\,(1 + \frac{a^2}{2}\sin^2\theta + \frac{3}{8}a^4\sin^4\theta + \frac{15}{24}a^6\sin^6\theta +)$$

Substituting for the powers of sin θ in the parenthesis their values in multiples of the angle, we have

$$\frac{\sin(\theta+\varphi)}{\cos\varphi} = \sin\theta + \frac{a}{2}\sin 2\theta \left| \begin{array}{l} 1 + \frac{a^2}{4} + \frac{9\,a^4}{16} + \frac{150}{768}a^6 + \ldots. \\ -\cos 2\theta\left(\frac{a^2}{4} + \frac{12\,a^4}{16} + \frac{225\,a^6}{768} + ..\right) \\ +\cos 4\theta\left(\frac{3\,a^4}{16} + \frac{90\,a^6}{768} + ...\right) \\ -\cos 6\theta\left(\frac{15\,a^6}{768} + ...\right) \end{array} \right|$$

Performing the multiplication indicated and collecting the terms we have

$$\frac{\sin(\theta+\varphi)}{\cos\varphi} = \sin\theta + \left| \begin{array}{l} +\sin 2\theta\left(\frac{a}{2} + \frac{a^3}{8} + \frac{15}{64}a^5 + ...\right) \\ -\sin 4\theta\left(\frac{a^3}{16} + \frac{12\,a^5}{64} + \frac{285}{3072}a^7 + \right) \\ +\sin 6\theta\left(\frac{3\,a^5}{64} + \frac{90\,a^7}{3072} + ...\right) \\ -\sin 8\theta\left(\frac{15\,a^7}{3072} + ...\right) \end{array} \right|$$

Therefore the turning moment due to a force P is

$$P r \left| \begin{array}{l} +\sin\theta \\ +\sin 2\theta \left(\frac{a}{2}+\frac{a^3}{8}+\frac{15\,a}{64}+\ldots\ldots\right) \\ -\sin 4\theta \left(\frac{a^3}{16}+\frac{12\,a^5}{64}+\frac{285}{3072}a^7+\right) \\ +\sin 6\theta \left(\frac{3\,a^5}{64}+\frac{90\,a^7}{3072}+\ldots\ldots\ldots\right) \\ -\sin 8\theta \left(\frac{15\,a^7}{3072}+\ldots\ldots\ldots\ldots\right) \end{array} \right| \qquad (3)$$

To obtain the turning moment due to the steam forces substitute for P the pressures on the piston at the different crank angles.

To obtain the turning moment due to the inertia of the reciprocating parts other than the connecting rod, substitute for P the mass of the reciprocating parts multiplied by the acceleration obtained from equation (2) as follows:

$$\frac{M}{g}\omega^2 r^2 \left| \begin{array}{l} +\sin\theta \left(\frac{a}{4}+\frac{a^3}{16}+\frac{15\,a^5}{128}+\ldots\ldots\right) \\ -\sin 2\theta \left(\frac{1}{2}+\frac{a^4}{16}+\frac{7\,a^6}{64}+\frac{6\,a^8}{256}+\right) \\ -\sin 3\theta \left(\frac{3\,a}{4}+\frac{5\,a^3}{16}+\frac{39\,a^5}{128}+\ldots\right) \\ -\sin 4\theta \left(\frac{a^2}{4}+\frac{3\,a^4}{16}+\frac{23\,a^5}{256}+\ldots\ldots\right) \\ +\sin 5\theta \left(\frac{a^3}{8}+\frac{3\,a^5}{16}+\ldots\ldots\ldots\right) \\ +\sin 6\theta \left(\frac{a^4}{16}+\frac{a^6}{8}+\frac{6\,a^8}{128}+\ldots\ldots\right) \end{array} \right| \qquad (4)$$

To obtain the inertia forces of the connecting rod we proceed as follows:

The rod is acted upon by three forces designated by F_1, F_2, F_3.

F_1 acts at the center of gravity of the rod in a direction parallel to the piston; F_2 acts along the rod towards the wrist pin, and F_3 acts at the center of percussion of the rod and at right angles to it as indicated in Fig. 2.

E, G and H, representing the wrist pin, center of gravity and center of percussion, respectively. G and H are distant d and λ from E.

The kinematical equations determining the motion of the rod are

$$F_1 = \frac{d^2 z}{dt^2}$$

$$F_2 = d \left(\frac{d\varphi}{dt}\right)^2$$

$$F_3 = d \frac{d^2 \varphi}{dt^2}$$

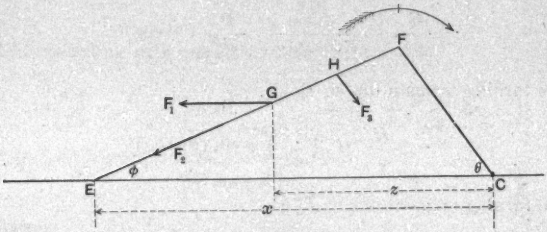


Fig. 2.

in which z, d and φ are respectively the abcissa of the center of gravity measured from the crank shaft, the distance of the center of gravity from the point E, and the angle the connecting rod makes with the center line of the engine.

The dynamical equations of the above forces where R is the mass of the rod are

$$\frac{R}{g} \cdot \frac{d^2 z}{dt^2}$$

$$\frac{R}{g} \cdot d \cdot \left(\frac{d\varphi}{dt}\right)^2$$

$$\frac{R}{g} \cdot d \cdot \frac{d^2 \varphi}{dt^2}$$

The turning moment about the crank shaft of the dynamical force F_1 is

$$-\frac{R}{g}\cdot\frac{d^2z}{dt^2}\cdot r\sin\theta$$

This force causes reactions at the guide and crank pin which are normal to the direction, and as the wrist pin is free to move in the direction of the force while the stub end of the rod is constrained by the crank pin, moments can be taken about the crank pin, and as the force passes through the center of gravity they are equal. The reaction on the crank pin is, therefore,

$$\frac{R}{g}\cdot\frac{(l-d)\sin\varphi}{l\cos\varphi}\cdot\frac{d^2z}{dt^2}$$

The turning moment due to this reaction is

$$-\frac{R}{g}\cdot\frac{(l-d)\sin\varphi}{l\cos\varphi}\cdot\frac{d^2z}{dt^2}\cdot r\cos\theta$$

The turning moment due to F_2 is

$$-\frac{R}{g}\cdot d\cdot\left(\frac{d\varphi}{dt}\right)^2 r\sin(\theta+\varphi)$$

The turning moment due to F_3 is

$$\frac{R}{g}\cdot d\cdot\frac{d^2\varphi}{dt^2}\cdot\frac{\lambda}{l}\cdot r\cos(\theta+\varphi)$$

The turning moment due to the connecting rod is the sum of

$$-\frac{R}{g}\cdot\frac{d^2z}{dt^2}\,r\sin\theta \tag{5}$$

$$-\frac{R}{g}\cdot\frac{(l-d)\sin\varphi}{l\cos\varphi}\cdot\frac{d^2z}{dt^2}\cdot r\cos\theta \tag{6}$$

$$-\frac{R}{g}\cdot d\cdot\left(\frac{d\varphi}{dt}\right)^2 r\sin(\theta+\varphi) \tag{7}$$

$$\frac{R}{g}\cdot d\cdot\frac{d^2\varphi}{dt^2}\cdot\frac{\lambda}{l}\cdot r\cos(\theta+\varphi) \tag{8}$$

The geometry of the figure gives

$$z = x - d\cos\varphi\,;\ \text{and calling}\ \frac{d}{l} = K,$$

we have

$$z = x - \frac{K r}{a}(1 - a^2\sin^2\theta)^{\frac{1}{2}}$$

$$z = x - \frac{K r}{a}\left|\begin{array}{l} 1 - \frac{a^2}{4} - \frac{3a^4}{64} - \frac{10a^6}{256} - \ldots\ldots\ldots \\ +\cos 2\theta\left(\frac{a^2}{4} + \frac{a^4}{16} + \frac{15a^6}{256} + \ldots\right) \\ -\cos 4\theta\left(\frac{a^4}{64} + \frac{6a^6}{256} + \ldots\ldots\ldots\right) \end{array}\right| \qquad (9)$$

Differentiating with respect to time

$$\frac{dz}{dt} = \frac{dx}{dt} - \frac{K r\omega}{a}\left|\begin{array}{l} -\sin 2\theta\left(\frac{a^2}{2} + \frac{a^4}{8} + \frac{15a^6}{128} + \ldots\right) \\ +\sin 4\theta\left(\frac{a^4}{16} + \frac{3}{32}a^6 + \ldots\ldots\ldots\right) \end{array}\right| \qquad (10)$$

Again,

$$\frac{d^2z}{dt^2} = \frac{d^2x}{dt^2} - \frac{K r\omega^2}{a}\left|\begin{array}{l} -\cos 2\theta\left(a^2 + \frac{a^4}{4} + \frac{15a^6}{64} + \ldots\right) \\ +\cos 4\theta\left(\frac{a^4}{4} + \frac{3}{8}a^6 + \ldots\ldots\ldots\right) \end{array}\right| \qquad (11)$$

Substituting for

$$\frac{d^2x}{dt^2}$$

its value from equation (2)

$$\frac{d^2z}{dt^2} = \omega^2 r\left|\begin{array}{l} -\cos\theta \\ (K-1)\left\{\begin{array}{l} \cos 2\theta\,(a + \frac{a^3}{4} + \frac{15a^5}{64} + \ldots) \\ -\cos 4\theta\left(\frac{a^3}{4} + \frac{3a^5}{8} + \ldots\right) \end{array}\right\} \end{array}\right| \qquad (12)$$

Multiplying the above by $r \sin \theta$, we have

$$\frac{d^2 z}{dt^2} \cdot r \sin \theta = \omega^2 r^2 \begin{vmatrix} \sin\theta\,(1-K)\left(\frac{a}{2}+\frac{a}{8}+\frac{15}{128}a^5+\ldots.\right) \\ -\sin 2\theta\,(\tfrac{1}{2}) \\ -\sin 3\theta\,(1-K)\left(\frac{a}{2}+\frac{5a^3}{32}+\frac{39}{128}a^5+\right) \\ +\sin 5\theta\,(1-K)\left(\frac{a^3}{8}+\frac{3a^5}{16}+\ldots.\right) \end{vmatrix} \quad (13)$$

Equation (6) may be written

$$-\frac{R}{g} \cdot \frac{d^2 z}{dt^2} r \sin\theta\,(1-K)\frac{a\cos\theta}{\cos\varphi}$$

which we see is equation (13) multiplied by

$$(1-K)\frac{a\cos\theta}{\cos\varphi}, \text{ omitting } \frac{R}{g}.$$

The geometry of the figure gives $\sin\varphi = a \sin\theta$

Differentiating we get $\cos\varphi\, d\varphi = a\cos\theta\, d\theta$, or

$$\frac{d\varphi}{d\theta} = \frac{a\cos\theta}{\cos\varphi} = a\cos\theta\,(1-a^2\sin^2\theta)^{-\frac{1}{2}}$$

Expanding the quantity in parenthesis we have

$$\frac{d\varphi}{d\theta} = a\cos\theta\,(1+\tfrac{1}{2}a^2\sin^2\theta+\tfrac{3}{8}a^4\sin^4\theta+\ldots.)$$

Integrating

$$\varphi = a\sin\theta+\frac{1}{6}a^3\sin^3\theta+\frac{3}{40}a^5\sin^5\theta+\ldots. \quad (14)$$

Transforming

$$\varphi = \sin\theta\,(a+\frac{a^3}{8}+\frac{3}{64}a^5+\ldots.)$$

$$- \sin 3\theta \left(+ \frac{a^3}{24} + \frac{3}{128} a^5 + \ldots.\right) \qquad (15)$$

$$+ \sin 5\theta \left(+ \frac{3}{640} a^5 + \ldots.\right)$$

Differentiating and calling the coefficients of $\sin \theta$, $\sin 3\theta$, $\sin 5\theta$, A_1, B_1 and C_1, respectively, we have

$$\frac{d\varphi}{d\theta} = \cos\theta\, A_1 - \cos 3\theta\, 3\, B_1 + \cos 5\theta\, 5\, C_1$$

Multiplying equation (13) by the above and again by $(1 - K)$ we have

$$\omega^2 r^3 \left| \begin{array}{l} - \sin\theta\,(1-K)\left(\frac{A_1 + 3B_1}{4}\right) \\ + \sin 2\theta\,(1-K)^2\left(\frac{K_1 A_1}{2} - \frac{K_2 A_1}{2} + \frac{3K_1 B_1}{2} - \frac{3K_3 B_1}{2} + \frac{5K_2 C_1}{2}\right) \\ - \sin 3\theta\,(1-K)\left(\frac{A_1}{4} + \frac{5C_1}{4}\right) \\ - \sin 4\theta\,(1-K)^2\left(\frac{A_1 K_2}{2} - \frac{A_1 K_3}{2} + \frac{3B_1 K_1}{2} + \frac{5C_1 K_1}{2}\right) \\ + \sin 5\theta\,(1-K)\,\frac{3B_1}{4} \\ + \sin 6\theta\,(1-K)^2\left(\frac{A_1 K_3}{2} + \frac{3B_1 K_2}{2} + \frac{5C_1 K_1}{2}\right) \\ - \sin 7\theta\,(1-K)\,\frac{5C_1}{2} \\ - \sin 8\theta\,(1-K)^2\left(\frac{3B_1 K_3}{2} + \frac{5C_1 K_2}{2}\right) \end{array} \right|$$

in which K_1, K_2 and K_3 are respectively the coefficients of $\sin\theta$, $\sin 3\theta$, $\sin 5\theta$ in equation (13).

Performing the operations indicated we have

$$\omega^2 r^2 (1-K)\begin{vmatrix} -\sin\theta\left(\frac{a}{4}+\frac{a^3}{16}+\frac{15}{492}a^5+\ldots.\right) \\ -\sin 2\theta(1-K)\left(-\frac{a^4}{64}+\frac{510}{7680}a^5\right) \\ -\sin 3\theta\left(\frac{a}{4}+\frac{a^3}{32}+\frac{9}{512}a^5\ldots\ldots\right) \\ -\sin 4\theta(1-K)\left(\frac{a^2}{4}+\frac{5a^4}{64}+\frac{448}{3072}a^5\right) \\ +\sin 5\theta\left(\frac{a^3}{32}+\frac{9}{512}a^5\ldots\ldots\ldots\right) \end{vmatrix} \tag{16}$$

Adding equations (13) and (16) we have

$$\frac{R}{g}\omega^2 r^2(1-K)\begin{vmatrix} +\sin\theta\left(\frac{a}{4}+\frac{a^3}{16}+\frac{45}{492}a^5\ldots\ldots\right) \\ -\sin 2\theta(1-K)\left(-\frac{a^4}{64}+\frac{510}{7680}a^5\right) \\ -\sin 3\theta\left(\frac{3a}{4}+\frac{3a^3}{16}+\frac{165}{512}a^5\ldots.\right) \\ -\sin 4\theta(1-K)\left(\frac{a^2}{4}+\frac{5a^4}{64}+\frac{448}{3072}a^5\right) \\ +\sin 5\theta\left(\frac{5a^3}{32}+\frac{105}{512}a^5\ldots\ldots\right) \end{vmatrix} - \frac{R}{g}\omega^2 r^2 \sin 2\theta \tfrac{1}{2} \tag{17}$$

which is the solution for the force F_1.

The development of the expression for the turning moment of the force F_2 is as follows:

From equation (15)

$$\frac{d\varphi}{dt} = \left[\cos\theta\left(a+\frac{a^3}{8}\right) - \cos 3\theta\,\frac{a^3}{8}\right]\omega$$

omitting the higher powers of a.

Squaring and omitting powers of a greater than 2 we have

$$\left(\frac{d\varphi}{dt}\right)^2 = a^2 \cos^2\theta\,\omega^2 \qquad (16)$$

and

$$\begin{aligned}\sin(\theta+\varphi) &= \sin\theta\cos\varphi + \cos\theta\sin\varphi\\ &= \sin\theta\,(1-a^2\sin^2\theta)^{\frac{1}{2}} + a\cos\theta\sin\theta\\ &= \sin\theta\left(1-\tfrac{1}{2}a^2\sin^2\theta-\tfrac{1}{8}a^4\sin^4\theta-\tfrac{1}{16}a^6\sin^6\theta-\right)\\ &\qquad + a\cos\theta\sin\theta\end{aligned}$$

therefore

$$\begin{aligned}\sin(\theta+\varphi) = \sin\theta&\left(1-\frac{3\,a^2}{8}-\frac{5\,a^4}{64}-\frac{55}{1024}a^6\right)\\ &+\sin 2\theta\left(\frac{a}{2}\right)\\ &+\sin 3\theta\left(\frac{a^2}{8}+\frac{5}{128}a^4+\frac{21}{1024}a^6\right)\\ &-\sin 5\theta\left(\frac{a^4}{128}+\frac{5\,a^6}{1024}\right)\\ &-\sin 7\theta\left(\frac{a^6}{1024}\right)\end{aligned}$$

Omitting terms containing powers of a greater than the first

$$\sin(\theta+\varphi) = \sin\theta + \sin 2\theta\left(\frac{a}{2}\right) \qquad (17)$$

Multiplying (16) and (17) we have

$$\begin{aligned}\left(\frac{d\varphi}{dt}\right)^2\sin(\theta+\varphi) &= \left[a^2\sin\theta\,(1-\sin^2\theta)\right.\\ &\qquad \left.+\frac{a^3}{2}\sin 2\theta\,(1-\sin^2\theta)\right]\omega^2\\ &= \frac{\omega^2 a^2}{4}\left|\begin{array}{l}+\sin\theta.\\ +\sin 2\theta.\,(a)\\ +\sin 3\theta.\\ +\sin 4\theta.\,\left(\frac{a}{2}\right)\end{array}\right|\end{aligned}$$

Multiplying the variable part of the moment by the constants we have

$$\frac{R}{g}\ \frac{\omega^2 . K . r^2 . a}{4}\ \left|\begin{array}{l} + \sin\theta \\ + \sin 2\theta\ (a) \\ + \sin 3\theta \\ + \sin 4\theta \left(\frac{a}{2}\right) \end{array}\right| \qquad (18)$$

The development of the turning moment due to F_3 is as follows:

$$\begin{aligned} \cos(\theta + \varphi) &= \cos\theta\cos\varphi - \sin\theta\sin\varphi \\ &= \cos\theta\,(1 - a^2\sin^2\theta)^{\frac{1}{2}} - a\sin^2\theta \\ &= -\frac{a}{2} + \cos\theta\left(1 - \frac{a^2}{8} - \frac{a^4}{64} - \frac{25}{1024}a^6\right) \\ &\quad + \cos 2\theta\left(\frac{a}{2}\right) \\ &\quad + \cos 3\theta\left(\frac{a^2}{8} + \frac{3}{128}a^4 + \frac{9}{1024}a^6\right) \qquad (19) \\ &\quad - \cos 5\theta\left(\frac{a^4}{128} + \frac{7\,a^6}{1024}\right) \\ &\quad - \cos 7\theta\left(\frac{a^6}{1024}\right) \end{aligned}$$

From equation (15) we have

$$\frac{d^2\varphi}{dt^2} = \omega^2\ \left|\begin{array}{l} - \sin\theta\,(a + \frac{a^3}{8} + \frac{3}{64}a^5 + \ldots\ldots) \\ + \sin 3\theta\,(+\frac{3}{8}a^3 + \frac{27}{128}a^5 + \ldots) \\ - \sin 5\theta\,(+\frac{75}{640}a^5 + \ldots\ldots) \end{array}\right| \qquad (20)$$

Multiplying (19) and (20) together we have

$$\frac{d^2\varphi}{dt^2}\cos(\theta+\varphi) = \frac{\omega^2}{2}\left|\begin{array}{l} +\sin\theta\left(\frac{3a^2}{2}+\frac{3}{8}a^4+\frac{45}{256}a^6\right) \\ +\sin 2\theta\left(-a+\frac{a^3}{2}+\frac{24}{128}a^5\right) \\ -\sin 3\theta\left(\frac{a^2}{2}+\frac{7a^4}{16}+\frac{375}{1280}a^6\right) \\ +\sin 4\theta\left(\frac{a^3}{4}+\frac{5a^5}{640}\right) \\ +\sin 5\theta\left(\frac{3a^4}{16}+\frac{285}{1280}a^6\right) \\ -\sin 6\theta\left(\frac{45a^5}{640}\right) \\ -\sin 7\theta\left(\frac{75a^6}{1280}\right) \end{array}\right| \quad (21)$$

Neglecting the higher powers of a

$$\frac{d^2\varphi}{dt^2}\cos(\theta+\varphi) = -\frac{\omega^2 a}{2}\sin 2\theta \quad (22)$$

Supplying the constants in equation (8) we have

$$+\frac{R}{g}\cdot d\cdot\frac{\lambda}{l}\cdot r\cdot\frac{\omega^2 a}{2}\sin 2\theta \quad (23)$$

Calling the moment of inertia of the rod about the wrist pin I, we substitute it for $R\,d\,\lambda$ and have finally for the turning moment due to the force F

$$\frac{I\ \omega^2 a^2}{2g}\sin 2\theta \quad (24)$$

The moment of inertia about a parallel axis is $I = I_c + d^2 R$ where I_c is the moment of inertia about the center of gravity.

Equation (24) may therefore be written

$$\frac{I\,\omega^2\,a^2}{2\,g}\sin 2\,\theta = \frac{I_c\,\omega^2\,a^2}{2\,g}\sin 2\,\theta + \frac{R\,d^2\,\omega^2\,a}{2\,g}\sin 2\,\theta$$

$$= I_c\,\frac{\omega^2\,a^2\sin 2\,\theta}{2\,g} + \frac{R\,K^2\,\omega^2\,r^2}{2\,g}\sin 2\,\theta$$

The turning moments due to F_1, F_2 and F_3 are:
for F_1

$$\frac{R}{g}\omega^2 r^2(1-K)\left|\begin{array}{l} +\sin\theta\left(\frac{a}{4}+\frac{a^3}{16}+\frac{45\,a^5}{492}\right) \\ -\sin 2\,\theta\,(1-K)\left(-\frac{a^4}{64}+\frac{510\,a^6}{7680}\right) \\ -\sin 3\,\theta\left(\frac{3\,a}{4}+\frac{3\,a^3}{16}+\frac{165\,a^5}{512}\right) \\ -\sin 4\,\theta\,(1-K)\left(\frac{a^2}{4}+\frac{5\,a^4}{64}+\frac{448\,a^6}{3072}\right) \\ +\sin 5\,\theta\left(\frac{5\,a^3}{32}+\frac{105\,a^5}{512}\right) \end{array}\right| - \frac{R}{g}\omega^2 r^2 \sin 2\theta\tfrac{1}{2}$$

$$\text{for } F_2, -\frac{R}{g}\cdot\frac{\omega^2\,K\,r^2\,a}{4}\left|\begin{array}{l} +\sin\theta \\ +\sin 2\,\theta\,(a) \\ +\sin 3\,\theta \\ +\sin 4\,\theta\left(\frac{a}{2}\right) \end{array}\right|$$

$$\text{for } F_3,\quad I_c\,\frac{\omega^2\,a^2\sin 2\,\theta}{2\,g} + \frac{R\,K^2\,\omega^2\,r^2\sin 2\,\theta}{2\,g}$$

The turning moment of the rod is the sum of the separate moments. Neglecting the term $\sin 2\,\theta$ in the parenthesis, for the turning moment of F_1, adding the term outside to the second term of F_2, and rearranging the terms in F_2, we have finally:

$$\frac{R}{g} \cdot \omega^2 r^2 (1 - K) \left| \begin{array}{l} + \sin \theta \left(\frac{a}{4} + \frac{a^3}{16} + \frac{45\, a^5}{492} \right) \\ - \sin 2\theta \left(\frac{1 + K}{2} \right) \\ - \sin 3\theta \left(\frac{3\, a}{4} + \frac{3\, a^3}{16} + \frac{165\, a^5}{512} \right) \\ - \sin 4\theta (1 - K) \left(\frac{a^3}{4} + \frac{5\, a^4}{64} + \frac{448\, a^6}{3072} \right) \\ + \sin 5\theta \left(\frac{5\, a^5}{32} + \frac{105\, a^5}{512} \right) \end{array} \right|$$

$$- \frac{R'}{g} \cdot \omega^2 r^2 (1 - K) \left| \begin{array}{l} + \sin \theta \, \frac{K a}{4 (1 - K)} \\ + \sin 2\theta \left(\frac{K a^2}{4 (1 - K)} \right) \\ + \sin 3\theta \, \frac{K a}{4 (1 - K)} \\ + \sin 4\theta \, \frac{K a^2}{8 (1 - K)} \end{array} \right| + I_c \frac{\omega^2 a^2 \sin 2\theta}{2 g} \qquad (25)$$

Part II.

The apparatus employed was a tuning fork electrically driven by a storage battery and a paper band secured to the face of the fly-wheel of the tandem engine before mentioned. Description of apparatus employed is as follows:

The Tuning Fork.—The tuning fork was made by Queen & Co. and was fitted with phosphor bronze fingers in order to increase the amplitude. It was driven by a storage battery and the usual make and break device employed. The period of the fork without the fingers was 100 per second, and with the fingers when in contact with the paper band 90. Care was taken in adjusting the fingers that the points would describe the same line on the paper when the wheel was revolved and the fork at rest.

To Further Explain.—When the fork is at rest and the wheel revolved the points described the line A B, and when the fork vibrated the points described the lines C D and E F. Assuming the fork's period constant, the distance between intersections 1 and 3 is the space travelled by the wheel in one complete vibration of the fork. The amplitude of the vibrations was 7 mms. The fork was mounted upon a slide rest, which was securely fastened to the floor. The fork, therefore, could be moved across the face of the wheel, the slide being in parallelism with the face of the wheel.

The Paper Band.—In order to have a smoother surface for the fingers of the fork to trace the curves and to facilitate the measurements, manila paper with a smooth surface was used. The paper may be obtained in rolls 12″ or more wide and of considerable length. The paper was secured to the face of the wheel by shellac varnish applied at the edges, and the joint was scarfed and secured by shellac varnish. Considerable care was

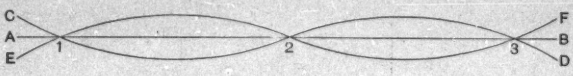


FIG. 3.

taken to avoid wrinkles. The paper was then slightly moistened with water applied with a paint brush. On drying it was found the surface was without wrinkles and smooth. Zinc paint, made by mixing zinc oxide ground in oil with spirits of turpentine, was applied to the paper with a paint brush. It is well known that silver, copper and brass will mark upon a surface coated as above.

After the wheel was prepared as described, the engine was put on its head center, corresponding to $\theta = 0$ in the theoretical part, and the tuning fork moved across the face, the points of the fingers describing two lines. This fixed the positions of all the tuning fork spaces relative to the crank positions.

THE TEST.

An artificial full load was put upon the generator by means of a water rheostat which consisted of iron plates immersed in a dock. Owing to the large amount of water the load was con-

stant. The ampere flow was 1,440 at 562 volts. I have carried on the same plates 3,400 amperes at 550 volts. After the engine had been running some time at this and lower loads in order that it might be thoroughly warmed, simultaneous readings of amperes and volts, indicator diagrams and record by the tuning fork were taken.

The running weights of the tandem engine were estimated as follows:

Low pressure piston	1,900 lbs.
High " "	500 "
Crosshead	1,200 "
Connecting rod	1,900 "
Piston rod	1,000 "

Length of connecting rod between centers 126″.
Crank radius, 21″.

$$\frac{r}{l} = a = 1/6$$

Moment of inertia of rod about its center of gravity approximately $9.402\ R\ r^2$

$$K = 0.6$$

Substituting in equation (25)

We have for the turning moment of the rod

$$\frac{R}{g}\ \omega^2\ r^2\ [-\sin\theta\ 0.0082 - \sin 2\theta\ 0.1934$$

$$-\sin 3\theta\ 0.0755 - \sin 4\theta\ 0.00316]$$

Substituting in equation (4)

We have turning moment of the reciprocating parts

$$\frac{M}{g}\ \omega^2\ r^2 \left| \begin{array}{l} +\sin\ \theta\ 0.04204 \\ -\sin 2\theta\ 0.50000 \\ -\sin 3\theta\ 0.12648 \\ -\sin 4\theta\ 0.00695 \end{array} \right|$$

Speed of engine 96 R. P. M., or 10.05 radians per second; $\omega_s = 101$.

From the above data Table I has been constructed. Ordinates in inches on a scale of 20,000 pounds per inch are given for the reciprocating parts M and for the rod R. The resultant ordinate is also given, so it is possible to plot the curves of turning moments of the total inertia forces or the parts separately.

The steam pressures are obtained from the indicator diagrams Table II, and the forces acting along the piston rod are given in Table III.

Equation (3) gives the turning moment due to the steam forces, and Table IV is the part within the parenthesis.

The second column of Table V gives the ordinates of the steam turning moments which are obtained by multiplying the last column of Table III (P), with the last column of Table IV together with the crank radius and dividing the product by the scale used, 20,000 pounds per inch.

The third column of Table V is obtained from Table I, and the fourth column is the algebraic sum which is the ordinate of the curve plotted.

The Curves.

Curve A is the curve obtained from the plotted values, and represents according to proper scale which is given, the turning moment at crank shaft center, the tangential force at crank pin, the acceleration at crank pin, or the acceleration at fly-wheel rim.

Considering it as a curve of acceleration at fly-wheel rim and integrating the area intercepted between the mean line (or curve of no acceleration) the curve and ordinates and plotting the values the curve of velocity variation is constructed, curve B. Proceeding in the same manner curve C is constructed which is the space variation.

Tables VI, VII, VIII, IX, give the measured values of four curves, which constitute four consecutive revolutions. Column I contains the numbers of the spaces described by the tuning fork in one period. The spaces are numbered consecutively, beginning at the head center or "0" angle. Column II shows the measurements of the spaces. Column III gives the average value of three consecutive measurements, for example, the value of the first space is the mean of the sum of measurements of the fifty-sixth, first and second spaces. The value of the second space is the mean of the sum of the measurements of the first, second and third spaces, and so on.

Column IV is formed by adding the second average value to the first for the value opposite the second space, the third to the second for the third space, and so on. Column IV shows the actual distance traveled by the wheel for any number of periods.

Column V is formed by taking the mean of the sum of average values in Column III, and setting them down as in Column IV. Column V shows the space traveled by a wheel rotating uniformly for any number of periods.

Column VI is formed by taking the first differences between Columns IV and V, and shows the actual space variation between a wheel rotating with constant angular velocity and one rotating with variable angular velocity.

An inspection of the curves shows a space variation of about 11 mms., whereas the calculated space variation is 7 mms. I am not prepared to say that this difference is caused by errors or by some lost motion, probably, the former, possibly both.

A fork having greater amplitude and higher period marking upon a smoother surface would undoubtedly give more accurate results. If, however, the information of this most interesting problem has been increased by this contribution its object has been fulfilled.

In conclusion, I desire to express my thanks to Mr. J. M. S. Waring, who has assisted me, and to whose efforts and patience too much praise cannot be given. I desire to thank also Mr. Harte Cooke and others for advice and assistance.

TABLE I.

	15°		30°		45°		60°		75°		90°		105°		120°		135°		150°		165°	
	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−
$+ .042 \sin \theta$ . . .	.0108		.0210		.0298		.0363		.0405		.042		.0405		.0363		.0298		.0210		.0108	
$- .500 \sin 2\theta$ . .		.25		.433		.5		.433		.25			.25		.433		.5		.433		.25	
$- .127 \sin 3\theta$ . .		.09		.127		.09			.09		.127		.09					.09		.127		.09
$- .007 \sin 4\theta$ . .		.0061		.0061			.0061		.0061					.0061		.0061			.0061		.0061	
Totals	.0108	.3461	.0210	.5661	.0298	.59	.0424	.433	.1366	.25	.169		.3805	.0061	.4693	.0061	.5298	.09	.4601	.127	.2669	.09
Sums		.3353		.5451		.5602		.3906		.1134	.169		.3744		.4632		.4398		.4331		.1769	
$\frac{M\omega^2 r^2}{g} = \frac{44000}{20000}$		.736		1.10		1.235		.860		.25	.372		.826		1.02		0.966		.952		.390	
$- \sin \theta \ .0082$ . . .		.00212		.00410		.00578		.0071		.00792		.0082		.00792		.0071		.00578		.0041		.00212
$- \sin 2\theta \ .1984$ . .		.09670		.16730		.1934		.1673		.09670			.09670		.1673		.1934		.1673		.0967	
$- \sin 3\theta \ .0755$ . .		.05330		.07550		.0533			.0533		.0755		.0533					.05330		.0755		.0533
$- \sin 4\theta \ .0032$		.00270		.00270			.0027		.0027					.00270		.0027			.0027		.0027	
Totals		.15482		.24960		.25248	.0027	.1744	.0560	.10402	.0755	.0082	.1500	.01062	.1673	.0098	.1934	.05908	.1700	.0796	.0994	.05542
Sums		.15482		.24960		.25248		.1717		.04862	.0673		.13938		.1575		.13432		.0904		.04398	
$\frac{R\omega^2 r^2}{g} = \frac{18200}{20000}$		.141		.227		.230		.156		.044	.061		.127		.144		.122		.0823		.04	
$R + M$		.877		1.327		1.465		1.016		0.294	0.433		0.953		1.164		1.088		1 034		0.43	

TABLE II.

Crank positions	0	1	2	3	4	5	6	7	8	9	10	11	12
	24	23	22	21	20	19	18	17	16	15	14	13	
H. P. Head . . .	130	127	126	123	122	98	73	61	51	44	41	38	33
	114	84	46	22	13	14	15	15	16	17	19	25	
H. P. Crank	38	42	54	61	71	90	114	127	127	127	126	126	126
	22	20	17	17	17	16	15	15	19	33	54	88	
L. P. Head	10	15	15	14.75	14.5	11.75	5.5	2.5	0.5	— 1.0	— 2.25	— 4.0	— 5.0
	1	— 3.75	— 8.25	— 8.25	— 8.25	— 8.0	— 8.0	— 8.0	— 8 0	— 8.0	— 7.75	— 7.0	
L. P. Crank	— 4.5	— 3.25	— 1.75	0	2.0	5.0	10.75	12.5	14.0	14.25	14.50	12.0	9.0
	— 6.0	— 7.0	— 8.0	— 8.5	— 8.5	— 8.75	— 9.0	— 9.0	— 9.0	— 9.0	— 9 0	— 7.0	

SHEET No. 1.

Tandem Compound Engine.

24–45–42.

Weight fly-wheel rim	56,000 lbs.	Radius of gyration	7,584 ft.
" " spokes	17,100 "	" "	4.500 "
" crank disc	8,000 "	" "	2,000 "
" armature	45,000 "	" "	3,000 "

Stroke of engine, 42 ins.

The above weights reduced to the crank center:

Fly-wheel rim	1,070,000 lbs.
" spokes	113,000 "
Crank disc	10,450 "
Armature	132,500 "
Total	1,325,950 lbs.

Curve No. 1.

Horizontal scale, $1'' = 60 \div (100 \times 24) = 0.025$ secs.

" " $1'' = (3.5 \times \pi) \div 24 = 0.460$ ft.

Turning moment at crank shaft center:

Vertical scale, $1'' = 20{,}000$ ft. lbs.

Tangential force at crank pin:

Vertical scale, $1'' = 20{,}000 \div 1.75 = 11{,}420$ lbs.

Acceleration at crank pin:

Vertical scale, $1'' = (11{,}420 \times 32.2) \div 1{,}325{,}950 = 0.278$ ft. per sec.

Acceleration at fly-wheel rim:

Vertical scale, $1'' = (0.278 \times 16) \div 3.5 = 1.27$ ft. per sec.

Areas, 1 sq. in. $= 11{,}420 \times 0.46 = 5{,}260$ ft. lbs.

Curve No. 2

Vertical scale, $1'' = 1.27 \times 2 \times 0.025 = 0.0635$ ft. per sec. at fly-wheel rim.

Maximum ordinate, 4.44 ins.

Totol velocity variation, $4.44 \times .0635 = 0.282$ ft. per sec.

Mean velocity, 88.75 ft. per sec.

Percentage of mean velocity, 0.00337.

Coefficient of unsteadiness, 296.

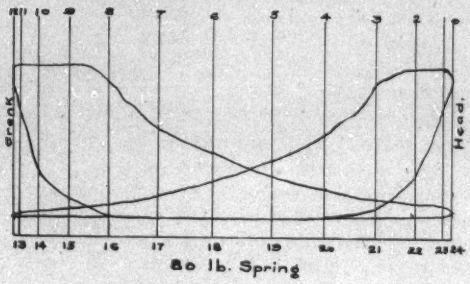
Curve No. 3.

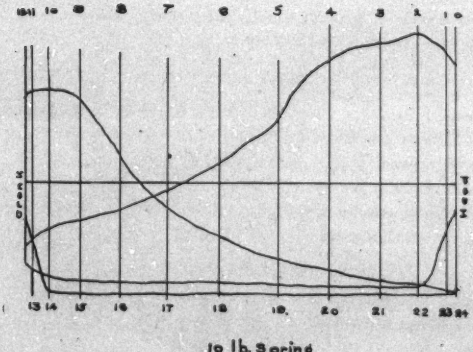
Vertical scale, $1'' = 0.0635 \times 2 \times .025 = 0.00317$ ft. at fly-wheel rim.

Maximum ordinate, 5.135 ins.

Total space variation, $5.135 \times .00317 = 0.0162$ ft.
$= 0.1945$ ins.

Crank pos.	0	1	2	3	4	5	6	7	8	9	10	11	12
" "	24	23	22	21	20	19	18	17	16	15	14	13	12
L.P Head	10¾	12	13	12	10¾	5¾	1½	-1	-2½	-3½	-4	-6	-6¾
" "	-2¾	-4¾	-9	-9	-9	-9	-9	-9	-9	-8½	-8	-7½	-7
" Crank	-9¾	-9¾	-9	-8¾	-7¾	-6½	-4½	-2	3	7½	8	8	8
" "	-9¾	-10	-10	-10	-10	-10	-10	-10	-10	-10	-9½	-5	-3
H.P Head	114	120	120	108	73.5	55	41.6	31.2	24	20.8	18.2	16.8	16
" "	10.8	9.8	4.72	22.4	14.4	14.4	16	16	16	16	16	16	16
" Crank	22.4	23.2	22.5	22.6	34.4	44.4	62.2	78.2	112	120	120	120	120
" "	18.4	16.8	16.0	16.0	15.2	14.4	14.4	14.4	14.4	22	46.4	92	106





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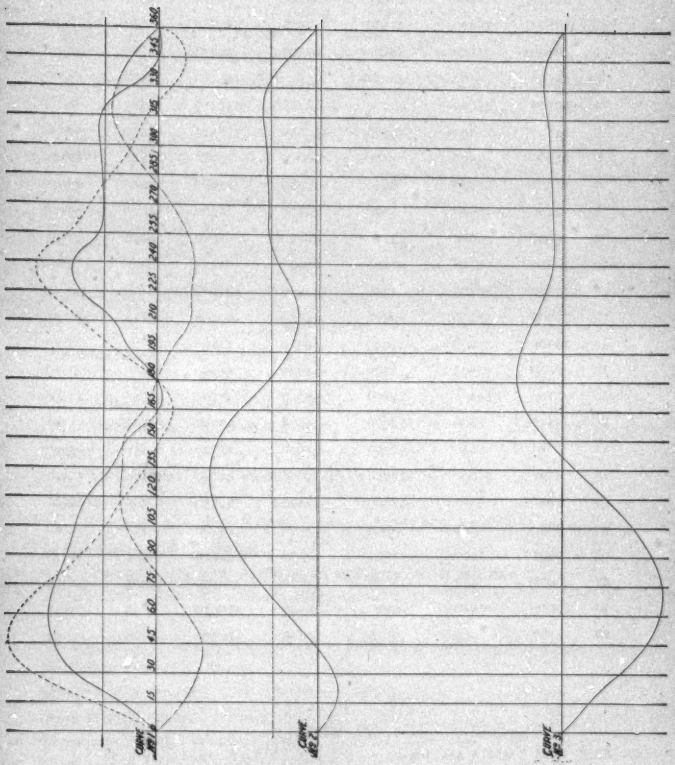
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345
330
315
300
285
270
255
240
225
210
195
180
165
150
135
120
105
90
75
60
45
30
15
CRANK No. 1
CRANK No. 2
CRANK No. 3

Sheet No. 1.

TABLE III.

	H.P.Head	H.P.Crank	L.P.Head	L.P.Crank	H.P.	L.P.	Combined
0	58900	16600	15745	-7030	49300	25120	74420
1	57500	18350	23600	-5075	46760	34530	83290
2	57000	23600	23600	-2735	49570	36100	85670
3	55600	26600	23200	0	48170	36500	84670
4	55200	31000	22900	3123	47770	36290	83970
5	44400	39300	16500	7828	37410	32200	69610
6	33000	49800	8675	16800	26430	22735	49165
7	27600	55500	3940	19550	21080	18000	39080
8	23100	55500	790	21900	14800	14850	29650
9	19900	55500	-1575	22280	5500	12485	17985
10	18550	55100	-3550	22680	-5030	10510	5480
11	17200	55100	-6310	18750	-21200	4620	-16580
12	14900	55100	-8675	14070	-40200	-22745	-62945
13	11300	38400	-11000	-10930	43800	29750	73550
14	8600	23600	-12200	-14060	46500	34850	81350
15	7700	14400	-12600	-14060	47800	34880	82680
16	7240	8300	-12600	-14060	48260	34500	82760
17	6790	6550	-12600	-14060	48710	32150	80860
18	6790	6550	-12600	-14060	43010	29400	72410
19	6530	6990	-12600	-13700	32970	20420	53390
20	5680	7430	-13000	-13300	25120	16123	41243
21	9950	7430	-13000	-13300	16650	13000	29650
22	20800	7430	-13000	-12500	2800	10265	13065
23	38000	8740	-6920	-10930	-19650	845	-18805
24	51600	9600	1575	-9375	-35000	-8605	-43605

Net Areas

H. P. Head 452.4 sq. ins. L. P. Head 1574.53

H. P. Crank 436.5 " " L. P. Crank 1562.16

SHEET No. 2.

TANDEM COMPOUND ENGINE.

24–45–42.

Weight fly-wheel rim	56,000 lbs.	Radius of gyration	7,584 ft.
" " spokes	17,100 "	" "	4,500 "
" crank disc	8,000 "	" "	2,000 "
" armature	45,000 "	" "	3,000 "

Stroke of engine, 42 ins.

The above weights reduced to the crank center:

Fly-wheel rim	1,070,000 lbs.
" spokes	113,000 "
Crank disc	10,450 "
Armature	132,500 "
Total	1,325,950 lbs.

CURVE "A."

Horizontal scale, $1'' = 60 \div (96 \times 24) = 0.026$ secs.

" " $1'' = (3.5 \times \pi) \div 24 = 0.460$ ft.

Turning moment at crank shaft center:

Vertical scale, $1'' = 20{,}000$ ft. lbs.

Tangential force at crank pin:

Vertical scale, $1'' = 20{,}000 \div 1.75 = 11{,}420$ lbs.

Acceleration at crank pin:

Vertical scale, $1'' = (11{,}420 \times 32.2) \div 1{,}325{,}950 = 0.278$ ft. per sec.

Acceleration at fly-wheel rim:

Vertical scale, $1' = (0.278 \times 16) \div 3.5 = 1.27$ ft. per sec.

CURVE "B."

Vertical scale, $1'' = 1.27 \times 2 \times 0.026 = 0.066$ ft. per sec. at fly-wheel rim.

Maximum ordinate, 5.84 ins.

Total velocity variation, $5.84 \times 0.066 = 0.385$ ft. per sec.

Mean velocity, 80.4 ft. per sec.

Percentage of mean velocity, 0.00479.

Coefficient of unsteadiness, 209.

CURVE "C."

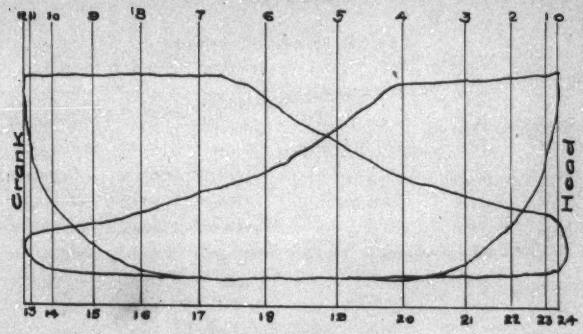
Vertical scale, $1'' = 0.066 \times 2 \times 0.026 = 0.00343$ ft. at fly-wheel rim.

Maximum ordinate, 6.67 ins.

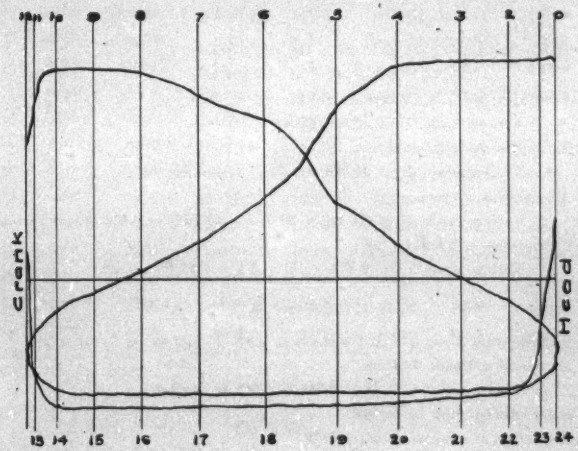
Total space variation, $6.67 \times 0.00343 = 0.0229$ ft.

$= 0.2748$ ins.

$= 6.9750$ mms.

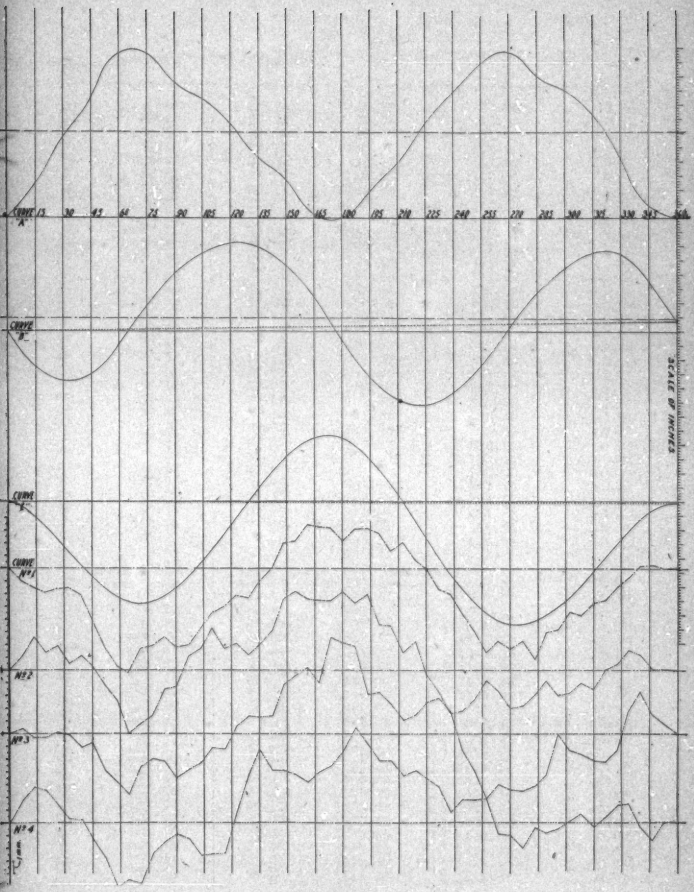


80 lb. Spring



10 lb. Spring

Sheet No. 2.



Sheet No. 2.

TABLE IV.

Angle.	$\sin \theta$	$\sin 2\theta \frac{a}{2}$	$\sin \theta + \sin 2\theta \frac{a}{2}$
0	0.000	0.0000	0.0000
15	0.259	0.0416	0.3006
30	0.500	0.0721	0.5721
45	0.707	0.0833	0.7903
60	0.866	0.0721	0.9381
75	0.966	0.0416	1.0076
90	1.000	0.0000	1.0000
05	0.966	— 0.0416	0.9244
120	0.866	— 0.0721	0.7939
133	0.707	— 0.0833	0.6237
150	0.500	— 0.0721	0.4279
165	0.259	— 0.0416	0.2174
180	0.000	0.0000	0.0000

TABLE V.

Crank Positions.	Ordinates steam turning moments.	Ordinates inertia turning moments.	Ordinates resultant turning moment.
0	0.00	0.00	0.00
1	2.18	— 0.88	1.30
2	4.28	— 1.33	2.95
3	5.83	— 1.47	4.36
4	6.90	— 1.02	5.88
5	6.10	— 0.29	5.81
6	4.30	+ 0.43	4.76
7	3.15	+ 0.95	4.10
8	2.05	+ 1.17	3.22
9	0.98	+ 1.09	2.07
10	0.21	+ 1.03	1.24
11	— 0.32	+ 0.43	0.11
12	0.00	0.00	0.00
13	1.39	— 0.43	0.96
14	3.06	— 1.03	2.03
15	4.52	— 1.09	3.43
16	5.75	— 1.17	4.58
17	6.51	— 0 95	5.56
18	6.35	— 1.43	5.92
19	4.73	+ 0.29	5.02
20	3.40	+ 1.02	4.42
21	2.05	+ 1.47	3.52
22	0.65	+ 1.33	1.98
23	— 0.50	+ 0.88	0.38
24	0.00	0.00	0.00

* Ordinates in inches. Scale, 20,000 lbs. per inch.
Note.—Mean ordinate of resultant turning moment 3.06", by Simpson's Rule.

TABLE VI.

CURVE No. 1

I	II	III	IV	V	VI
0	27.30	26.96	26.96	27.06	-0.10
1	26.80	27.02	53.98	54.13	-0.15
2	26.98	27.02	81.00	81.19	-0.19
3	27.32	27.09	108.09	108.25	-0.16
4	26.99	27.08	135.17	135.32	-0.15
5	26.93	26.99	162.16	162.37	-0.21
6	27.05	26.87	189.03	189.44	-0.41
7	26.63	26.85	215.88	216.50	-0.62
8	26.88	26.92	242.80	243.57	-0.77
9	27.25	27.01	269.81	270.63	-0.82
10	26.90	27.25	297.06	297.69	-0.63
11	27.60	27.09	324.15	324.76	-0.61
12	26.78	27.15	351.28	351.82	-0.54
13	27.00	26.98	378.26	378.88	-0.62
14	27.15	27.09	405.35	405.95	-0.60
15	27.11	27.19	432.54	433.01	-0.47
16	27.30	27.18	459.72	460.07	-0.35
17	27.12	27.12	486.84	487.14	-0.30
18	26.93	27.15	513.99	514.20	-0.21
19	27.40	27.04	541.03	541.26	-0.23
20	26.78	27.20	568.23	568.33	-0.10
21	27.42	27.16	595.39	595.39	±0.00
22	27.28	27.26	622.65	622.45	+0.20
23	27.08	27.09	649.74	649.52	+0.22
24	26.92	27.19	676.93	676.68	+0.25
25	27.57	27.04	703.97	703.64	+0.33
26	26.62	27.06	731.02	730.90	+0.32
27	26.96	26.97	757.99	757.77	+0.22
28	27.32	27.15	785.14	784.83	+0.31
29	27.18	27.07	812.21	811.89	+0.32
30	26.70	27.03	839.24	838.96	+0.28
31	27.20	26.92	866.16	866.02	+0.14
32	26.85	27.13	893.29	893.08	+0.21
33	27.38	26.93	920.22	920.15	+0.07
34	26.58	26.99	947.21	947.21	±0.00
35	27.04	26.90	974.11	974.27	-0.16
36	27.08	27.02	1001.13	1001.33	-0.20
37	26.95	26.91	1028.04	1028.39	-0.35
38	26.70	26.93	1054.97	1055.46	-0.49
39	27.15	26.90	1081.87	1082.83	-0.68
40	26.85	27.15	1109.02	1109.59	-0.57
41	27.45	27.00	1136.02	1136.65	-0.63
42	26.70	27.12	1163.14	1163.71	-0.57
43	27.22	26.93	1190.07	1190.78	-0.71
44	26.88	27.27	1217.34	1217.84	-0.50
45	27.70	27.08	1244.42	1244.90	-0.48
46	26.65	27.21	1271.63	1271.97	-0.34
47	27.27	27.03	1298.66	1299.03	-0.37
48	27.18	27.16	1325.82	1326.09	-0.27
49	27.03	27.09	1352.91	1353.16	-0.25
50	27.07	27.17	1380.08	1380.22	-0.14
51	27.42	27.13	1407.21	1407.28	±0.07
52	26.90	27.16	1434.37	1434.35	+0.02
53	27.16	27.07	1461.44	1461.41	+0.03
54	27.16	27.03	1488.47	1488.47	±0.00
55	26.78	27.08	1515.55	1515.54	+0.01

TABLE VII

CURVE No. 2

I	II	III	IV	V	VI
0	27.25	27.18	27.18	27.07	+0.11
1	27.18	27.23	54.41	54.14	+0.27
2	27.25	26.94	81.35	81.21	+0.14
3	26.40	27.13	108.48	108.39	+0.19
4	27.73	26.93	135.41	135.36	+0.05
5	26.65	27.13	162.54	162.43	+0.11
6	27.00	26.99	189.53	189.50	+0.03
7	27.33	26.92	216.45	216.57	-0.12
8	26.42	26.95	243.40	243.64	-0.24
9	27.10	26.82	270.22	270.71	-0.50
10	26.95	27.14	297.36	297.79	-0.42
11	27.38	27.14	324.50	324.86	-0.36
12	27.10	27.28	351.78	351.93	-0.15
13	27.35	27.09	378.87	379.00	-0.13
14	26.82	27.31	406.18	406.07	+0.11
15	27.78	27.14	433.32	433.15	+0.17
16	26.65	27.23	460.55	460.22	+0.33
17	27.30	26.91	487.46	487.29	+0.17
18	26.78	27.11	514.57	514.36	+0.21
19	27.28	26.98	541.55	541.43	+0.12
20	26.90	27.15	568.70	568.50	+0.20
21	27.30	27.18	595.88	595.58	+0.31
22	27.35	27.32	623.20	623.55	+0.56
23	27.30	27.14	650.34	649.72	+0.62
24	26.78	27.01	677.35	676.79	+0.56
25	26.95	27.06	704.41	703.86	+0.55
26	27.45	27.07	731.48	730.93	+0.55
27	26.80	27.14	758.62	758.00	+0.62
28	27.18	26.99	785.61	785.08	+0.53
29	27.00	27.14	812.75	812.15	+0.60
30	27.23	26.84	839.59	839.22	+0.37
31	26.30	27.05	866.64	866.29	+0.35
32	27.62	26.86	893.50	893.36	+0.14
33	26.65	27.16	920.66	920.43	+0.23
34	27.20	26.82	947.48	947.58	-0.08
35	26.60	26.93	974.41	974.58	-0.17
36	27.00	26.90	1001.31	1001.65	-0.34
37	27.10	27.09	1028.40	1028.72	-0.32
38	27.17	27.16	1055.56	1055.79	-0.23
39	27.20	27.22	1082.78	1082.86	-0.08
40	27.28	26.99	1109.77	1109.94	-0.16
41	26.50	26.94	1136.71	1137.01	-0.30
42	27.05	27.09	1163.80	1164.08	-0.28
43	27.73	27.16	1190.96	1191.15	-0.19
44	26.70	27.18	1218.14	1218.22	-0.08
45	27.12	26.95	1245.09	1245.29	-0.20
46	27.03	27.10	1272.19	1272.37	-0.18
47	27.15	27.15	1299.34	1299.44	-0.10
48	27.28	27.02	1326.36	1326.51	-0.15
49	26.63	27.22	1353.58	1353.58	±0.000
50	27.75	27.12	1380.70	1380.65	+0.05
51	26.98	27.18	1407.88	1407.72	+0.16
52	26.80	27.03	1434.91	1434.79	+0.12
53	27.30	26.97	1461.88	1461.87	+0.01
54	26.80	27.07	1488.95	1488.94	+0.01
55	27.12	27.06	1516.01	1516.01	±0.000

TABLE VIII.

CURVE No.3

I	II	III	IV	V	VI
0	27.00	27.09	27.09	27.05	+0104
1	26.80	26.97	54.06	54,09	-0.03
2	27.12	27.06	81.11	81.14	-0.03
3	27.22	27.08	108.19	108.18	+0.01
4	26.70	27.03	135.21	135.23	-0.01
5	27.18	26.94	162.16	162.27	-0.11
6	26.95	27.09	189.25	189.32	-0.07
7	27.13	26.82	216.07	216.36	-0.29
8	26.37	26.96	243.03	243.41	-0.38
9	27.35	26.95	269.98	270.46	-0.48
10	27.10	27.26	297.24	297.50	-0.26
11	27.30	27.11	324.35	325.55	-0.20
12	26.93	27.02	351.37	351.59	-0.22
13	26.82	26.92	378.29	378.64	-0.35
14	27.00	27.11	405.40	405.68	-0.28
15	27.50	27.10	432.50	432.73	-0.23
16	26.80	27.17	459.67	459.76	-0.11
17	27.20	27.03	486.70	486.82	-0.12
18	27.08	27.23	513.90	513.87	+0.06
19	27.42	27.12	541.05	540.91	+0.14
20	26.86	27.04	568.09	567.96	+0.13
21	26.85	27.05	595.14	595.00	+0.14
22	27.45	27.30	622.44	622.05	+0.39
23	27.60	27.11	649.55	649.09	+0.46
24	26.27	27.11	676.66	676.14	+0.52
25	27.47	26.93	703.59	703.19	+0.40
26	27.08	27.40	730.99	730.23	+0.76
27	27.67	26.91	757.90	757.28	+0.72
28	26.00	27.01	784.91	784.32	+0.69
29	27.35	26.77	811.68	811.37	+0.31
30	26.97	27.06	838.74	838.41	+0.33
31	26.85	26.96	865.70	865.46	+0.24
32	27.05	26.91	892.61	892.50	+0.11
33	26.83	27.09	919.70	919.55	+0.15
34	27.40	27.15	946.85	946.60	+0.25
35	27.22	27.07	973.92	973.64	+0.28
36	26.60	26.95	1000.87	1000.69	+0.18
37	27.02	26.73	1027.60	1027.73	-0.13
38	26.58	26.88	1054.48	1054.78	-0.30
39	27.02	26.83	1081.31	1081.62	-0.31
40	26.90	27.16	1108.47	1108.87	-0.40
41	27.55	27.01	1135.48	1135.92	-0.44
42	26.58	27.03	1162.51	1162.96	-0.45
43	26.95	27.09	1189.60	1190.01	-0.41
44	27.75	27.13	1316.73	1217.05	-0.32
45	26.70	27.37	1244.10	1244.10	±0.000
46	27.65	26.91	1271.01	1271.14	-0.13
47	26.37	27.02	1298.03	1298.19	-0.16
48	27.05	26.99	1325.02	1325.23	-0.21
49	26.55	27.04	1352.06	1352.28	-0.22
50	27.32	27.14	1379.20	1379.33	-0.13
51	27.35	27.35	1406.55	1406.37	+0.18
52	27.17	27.19	1433.74	1433.42	+0.33
53	27.05	26.87	1460.61	1460.46	+0.15
54	26.40	26.98	1487.59	1487.50	+0.08
55	27.46	26.96	1514.55	1514.55	±0.00

TABLE IX.

CURVE No. 4

I	II	III	IV	V	VI
0	27.12	27.21	27.21	27.03	+0.18
1	27.12	27.13	54.34	54.06	+0.28
2	27.15	26.99	81.33	81.08	+0.25
3	26.70	26.92	108.25	108.11	+0.14
4	26.92	26.92	135.17	135.14	+0.03
5	27.13	27.02	162.19	162.17	+0.02
6	27.00	26.88	189.07	189.30	-0.13
7	26.52	26.87	215.94	216.22	-0.28
8	27.10	26.81	242.75	243.25	-0.50
9	26.80	27.08	269.83	270.28	-0.45
10	27.35	27.00	296.83	297.31	-0.48
11	26.85	27.27	324.10	324.34	-0.24
12	27.60	27.11	351.21	351.36	-0.15
13	26.88	27.09	378.30	378.39	-0.09
14	26.80	26.99	405.29	405.42	-0.13
15	27.30	26.90	432.19	432.45	-0.26
16	26.60	27.04	459.23	458.48	+0.25
17	27.22	27.03	486.26	486.50	-0.24
18	27.28	27.35	513.61	513.53	+0.08
19	27.33	27.33	540.94	540.56	+0.38
20	27.10	27.23	568.17	567.59	+0.58
21	27.00	26.96	595.13	594.62	+0.41
22	26.73	26.92	622.05	621.64	+0.41
23	27.03	27.00	649.05	648.67	+0.38
24	27.25	26.97	676.02	675.70	+0.32
25	26.63	27.11	703.13	702.73	+0.40
26	27.45	27.08	730.21	729.76	+0.45
27	27.13	27.13	757.34	756.78	+0.56
28	26.90	27.22	784.56	783.81	+0.75
29	26.70	26.90	811.46	810.84	+0.62
30	27.30	26.90	838.36	837.87	+0.49
31	26.90	27.03	865.39	864.90	+0.49
32	27.08	26.90	892.29	891.93	+0.37
33	26.82	27.07	919.36	918.95	+0.41
34	27.30	26.97	946.33	945.98	+0.35
35	26.78	26.96	973.29	973.01	+0.28
36	26.80	26.83	1000.12	1000.04	+0.08
37	26.90	27.12	1027.24	1027.06	+0.18
38	27.65	27.03	1054.27	1054.09	+0.18
39	26.85	27.04	1081.31	1081.12	+0.19
40	26.93	26.74	1108.05	1108.15	-0.09
41	26.75	27.03	1135.08	1135.18	-0.10
42	27.40	27.02	1162.10	1162.20	-0.20
43	26.90	27.09	1189.19	1189.23	-0.04
44	26.97	26.99	1216.18	1216.26	-0.08
45	27.10	27.06	1243.24	1243.29	-0.05
46	27.10	27.09	1270.33	1270.32	+0.01
47	27.08	27.08	1297.41	1297.34	+0.07
48	27.07	26.93	1324.34	1324.37	-0.03
49	26.65	27.10	1351.44	1351.40	+0.04
50	27.58	27.13	1378.57	1378.42	+0.14
51	27.15	27.04	1405.61	1405.46	+0.15
52	26.38	26.84	1432.45	1432.48	-0.03
53	27.00	26.92	1459.37	1459.51	-0.14
54	27.88	27.17	1486.54	1486.54	±0.000

TABLE X.

Crank angles....	o	o	15	15	30	30	45	45	60	60	75	75	90	90	105	105	120	120	135	135	150	150	165	165	180	180
	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−
+ .05 sin θ.....	o	o	.013		.025		.035		.044		.048		.05		.048		.044		.035		.025		.013		o	o
− .5 sin 2 θ	o	o		.25		.44		.50		.44		.25	o	o	.25		.44		.50		.44		.25		o	o
− .15 sin 3 θ ...	o	o		.105		.15		.105		o	.105		.15		.105		o	o		.105		.15		.105	o	o
− .01 sin 4 θ ..	o	o		.009		.009	o	o	.009		.009		o	o		.009		.009	o	o	.009		.009		o	o
Totals	o	o	.013	.364	.025	.599	.035	.605	.053	.440	.162	.250	.200	o	.403	.009	.484	.009	.535	.105	.474	.150	.272	.105	o	o
Sums	o	o		.351		.574		.570		.387		.088	.200		.394		.475		.430		.324		.167		o	o
$\frac{M\omega^2 r^2}{g} = \frac{57000}{17800}$				1.12″		1.84″		1.82″		1.24″		.28″	.64		1.26″		1.52″		1.38″		1.04″		.56″			
$\frac{M\omega^2 r^2}{g} = \frac{81000}{17800}$				1.59″		2.62″		2.60″		1.76″		.40″	.91		1.79″		2.16″		1.96″		1.48″		.76″			
− .0025 sin θ...				.0007		.0013		.0018		.0022		.0024		.0025		.0024		.0022		.0018		.0013		.0007		
− .296 sin 2 θ ..				.148		.258		.296		.258		.148			.148		.258		.296		.258		.148			
− .0975 sin 3 θ..				.069		.098		.069			.069		.098		.069					.069		.098		.069			
− .0047 sin 4 θ..				.0041		.0041			.0041		.0041					.0041		.0041			.0041		.0041			
Totals				.2218		.3614		.3668	.0041	.2609	.0731	.1504	.098	.0025	.217	.0065	.258	.0063	.296	.0708	.2621	.0998	.1521	.0697		
Sums				.2218		.3614		.3668		.2501		.0773	.0955		.2105		.2517		.2252		.1628		.0824			
$\frac{R\omega^2 r^2}{g} = \frac{52000}{17800}$				.66″		1.07″		1.08″		.77″		.23″	.28″		.62″		.74′		.67″		.48″		.24″			
$M + R$ High...				1.78″		2.91″		2.90″		2.01″		0.51′	0.92″		1.88″		2.26″		2 05″		1.52″		.80″			
$M + R$ Low ...				2 25″		3.69″		3.68″		2.53″		0.63″	1.19″		2.41		2.90″		2.63″		1. 6″		1.00″			
Crank angles....		o	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360
		+	−	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
$M + R$ High ..		o	1.78	2.91	2.90	2.01	0.51	0.92	1.88	2.26	2.05	1.52	.80	o	.80	1.52	2.05	2.26	1.88	0.92	0.51	2.01	2.90	2.91	1.78	o
$M + R$ Low...		−	+	+	+	+	+	−	−	−	−	−	−	+	+	+	+	+	+	−	−	−	−	−	−	−
		1.19	0.63	2 53	3.68	3.69	2.25	o	2.25	3.69	3.68	2.53	0.63	1.19	2.41	2.90	2.63	1.96	1.00	o	1.00	1.96	2.63	2.90	2.41	1.19
Gravity.....		−	−	−	−	+	+	+	+	+	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−	−
		0.78	0.60	0.39	0.14	0.37	0.67	0.79	0.42	1.05	0.99	0.91	0.78	1.59	0.35	0.12	0.15	0.39	0.67	0.78	0.90	1.03	0.96	0.90	0.78
Resultant inertia.		−	−	−	+	+	+	+	+	−	−	−	+	+	+	+	+	−	−	−	−	−	−	−	−	−
		1.97	1.75	0.77	0.64	1.80	2.11	1.59	0.42	0.51	0.58	0.02	1.08	1.97	2.20	1.73	0.70	0.45	1.27	1.59	1.27	0.85	0.76	0 95	1.52	1.97

SHEET No. 3.

Vertical Cross-Compound Engine.

29-60-56.

Curve No. 1.

Horizontal scale, $1'' = 60 \div (98.6 \times 48) = 0.0127$ secs.

" " $1'' = (4.66 \times \pi) \div 48 = 0.3055$ ft.

Turning moment at crank shaft center:

Vertical scale, $1'' = 17{,}800$ lbs.

Tangential force at crank pin:

Vertical scale, $1'' = 17{,}800 \div 2.33 = 7{,}640$ lbs.

Acceleration at crank pin:

Vertical scale, $1'' = (7{,}640 \times 32.2) \div 1{,}295{,}000 = 0.1888$ ft. per sec.

Acceleration at fly-wheel rim:

Vertical scale, $1'' = (0.1888 \times 108) \div 28 = 0.728$ ft. per sec.

Areas, 1 sq. in. $= 7{,}640 \times 0.3055 = 2{,}335$ ft. lbs.

Curve No. 2.

Horizontal scale, $1'' = 0.0127$ secs.

Vertical scale, $1'' = 0.728 \times 6 \times 0.0127 = 0.055$ ft. per sec. at wheel rim.

Maximum ordinate, 5.8 ins.

Total velocity variation, $5.8 \times 0.055 = 0.3215$ ft. per sec.

Percentage of total velocity variation, 0.00346.

Coefficient of unsteadiness, 289.

Curve No. 3.

Horizontal scale, $1'' = 0.0127$ secs.

Vertical scale, $1'' = 0.055 \times 6 \times 0.0127 = 0.00438$ ft. at wheel rim.

Maximum ordinate, 5.0 ins.

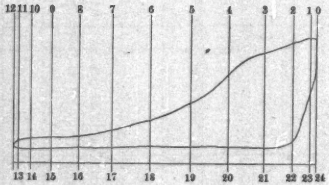
Total space variation, $5.0 \times 0.00438 = 0.0219$ ft.,

$= 0.263$ ins.

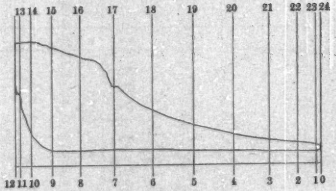
$= 0.1394$ degs.

STEAM BY GAUGE	137.5 LBS.
VACUUM	24.5 INS.
SPEED	98.6 R.P.M.
AMPERES	2300.
VOLTS	526.

0	1	2	3	4	5	6	7	8	9	10	11	12	
24	23	22	21	20	19	18	17	16	15	14	13		
5½	7	8	9	9½	10	10	10½	10½	10½	9½	6	4	L.P.C.
-5½	-4½	-3½	-2½	-1	2	7½	9½	11	12½	13	12½	5	" " "
4½	10½	10½	10	9	6½	1½	-1½	-3½	-4½	-5½	-6	-6½	L.P.H.
3	7	10	11	11	11	11	10½	10½	10½	10	9½	9	" " "
126	128	121	111	87	59	43	32	26	25	23	21	17	H.P.H.
102	75	21	16	15½	16	15	14	13	13	13	14	17	" " "
15	13	12	13	13	13½	14	14½	14½	14	31	71	93	H.P.C.
15	18	21	24	30	39	55	78	108	118	123½	123	123	" " "



60 LB. SPRING

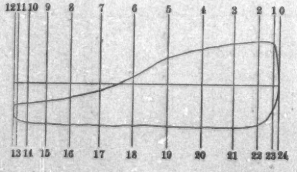


HEAD END

No.1 Engine – Mar. 11, 1901
Series No.9

CRANK END

Table XI.



20 LB. SPRING

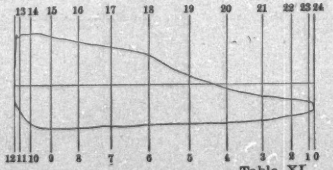
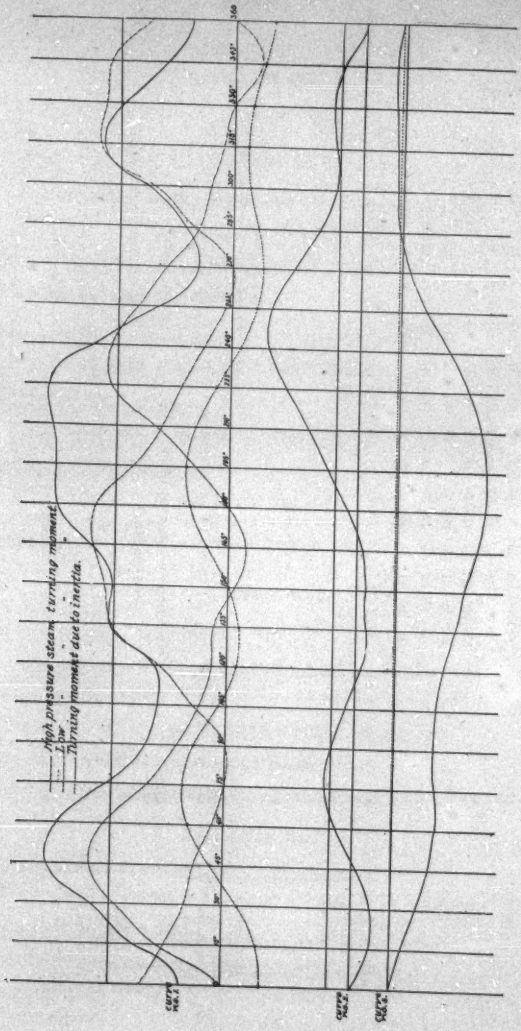


Table XI


High pressure steam turning moment
Low
Turning moment due to inertia.
0°
15°
30°
45°
60°
75°
90°
105°
120°
135°
150°
165°
180°
195°
210°
225°
240°
255°
270°
285°
300°
315°
330°
345°
360°
375°
390°
405°
420°
435°
450°
465°
480°
495°
510°
525°
540°
555°
570°
585°
600°
615°
630°
645°
660°
675°
690°
705°
720°
3275
3280
3285

Sheet No. 8.

A paper presented at the 157th Meeting of the American Institute of Electrical Engineers, New York, October 25th, 1901, President Steinmetz in the Chair.

ANGULAR VELOCITY IN STEAM ENGINES IN RELATION TO PARALLELING OF ALTERNATORS.

BY WALTER I. SLICHTER.

Certain characteristics of the steam engine which are of no importance in most applications of power other than electrical and even in electrical when D. C. generators are used, become of considerable importance when the engine is direct connected to an alternating current generator. These are the peculiar variations in tangential effort on the crank, and the consequent variation in speed of the shaft and fly-wheel.

The angular velocity of the fly-wheel varies in two entirely different ways, due to two different causes. The first and best known is that due to a change of load, then the average velocity changes, and there is a change in the number of revolutions per minute.

The second effect is due to irregularities in the applied force and transmission of that force, and consists in a variation of the instantaneous value of the angular velocity, while the average value and the revolutions per minute remain the same and constant.

The first is altogether a function of the governor of the engine. The latter phenomenon, which is the one to be discussed in this article, is more complicated both in its causes and effects, as it introduces the distribution of the steam pressure in the cylinder, inertia of reciprocating parts, angularity of connecting rod and moment of fly-wheel. It is of importance where two or more alternators are to be run in parallel, or where synchronous motors

are to be run from one or more alternators. Such machines tend to run in exact synchronism, and their inertia gives them a tendency to run at constant angular velocity. If now an irregular pulsation is impressed upon the system by the steam engine, there is a continual effort in each machine to keep in exact step with this pulsation, which causes an exchange of cross-currents and a liability of falling out of step.

When two alternators are running in parallel, if they differ in angular position, even if revolving at the same speed, a cross-current will flow between them, tending to pull them together. This cross-current may be wattless but even then it consumes some energy in the form of $I^2 r$.

Assuming that we wish to limit this cross-current to 10% of the full load current of one generator, what angular displacement will be allowable?

Taking several representative alternators having an average regulation of 9%, we find the average short-circuit current is 2.5 times full load current, thus the synchronous impedance is

$$z = e/i = E/2.5.$$

The difference between any two sine waves having an amplitude E and displaced in phase by an angle β is $2\ E \sin \beta/2$. This E. M. F. is short-circuited through the impedance of the two alternators in series.

$$\text{Thus cross-current} = e/z = \frac{2\ E \sin \beta/2}{2 \times E/2.5} = 2.5 \sin \beta/2$$

placing this equal to .10, we have $\sin \beta/2 = .04$, $\beta/2 = 2.3°$ displacement between one wave and mean.

If we say 2.5° displacement for round numbers, we have a cross-current of .109.

(A displacement of 2.5° in the electrical circuit is caused by a much less displacement of the fly-wheel. There is one complete cycle of E. M. F. generated per pair of poles. Thus if p is the number of pairs of poles on the generator, there are $p \times 360$ degrees of E. M. F. generated in one revolution of the fly-wheel. That is, one degree on the fly-wheel equals p degrees of E. M. F.)

The causes of the irregular effort are: The pressure of the steam on the piston due to cut off and expansion is uneven, being great during the first part of stroke, and small and even negative, due to compression, at the end. In the vertical engine the weight of the piston, rod, cross-head and part of connecting rod have to be lifted on the up-stroke, but are additive to the steam pressure on the down stroke. This makes a difference in

the power of each stroke, raises the peak of one and increases unbalanced energy.

This pressure is transferred to the crank-pin through a varying angle so the effort is roughly equal to P sin φ, varying between zero at beginning and end of stroke and P at middle. Thus there is a combination of maximum values at the point of cut-off, as that comes near the middle of the stroke, and we get quite a pointed peak here.

The angularity of the connecting rod has a different effect in the forward and backward strokes, as in the forward stroke during the early part while the pressure is high anyway, it increases the effect of that pressure on the crank pin, and in the latter part it diminishes that effect. On the return stroke it has the reverse effect, diminishing in the early part and increasing in the latter. So that the final effect is to increase the peak of one impulse and diminish the peak of the other.

The inertia of reciprocating parts absorbs energy at the beginning of the stroke, and gives it up at the end, so it has a beneficial effect on the diagram, smoothing the peak and raising the low values. The combination of two or more cylinders on the shaft naturally tends to even the effort and also to diminish the length of time one impulse lasts, thus diminishing its effect.

Combining the effort of three cylinders would seem to give a more even torque than two cylinders, but the actual diagrammatic combination shows there is very little choice between a two cylinder or three-cylinder engine.

In a single cylinder, double-acting engine there are two impulses per revolution, and in a cross-compound engine there are four impulses. Each impulse is more or less like any other. By analyzing the variation in speed during one impulse we can derive our conclusions. Though as all the impulses are not equal, it is as well to select that impulse having the greatest amount of unbalanced energy so as to know the worst conditions.

During the first quarter of impulse, the effort is less than average, during the second and third quarter (approximately) the effort is greater, and during the last quarter it is low again. The unbalanced effort represents a force which, acting on the mass of the fly-wheel, produces an acceleration, $a = f/m$. Thus by referring the torque diagram (Fig. 1.) to the line of average torque as base, we get a curve of acceleration.

Integrating this acceleration, we get a curve of velocity v.

This curve lags 90° behind the acceleration curve. Thus starting from a point of minimum torque a, the acceleration will be negative. From a to b the torque is less than average, the acceleration negative, the speed will keep dropping in value until b is reached, where, since the acceleration changes sign, v is a minimum, and commences to increase again; v will keep on increasing as long as the acceleration is positive, or to d, where v has its maximum value.

If we assume the variation is sinusoidal, then the average value of v from a to c is v_1, and of s, the percentage variation in speed, is $2\,s/\pi$.

The speed is low from a to c, or while the crank is passing $\theta°$ of the crank circle; during this time the crank continually falls behind the position it would maintain at constant angular veloc-

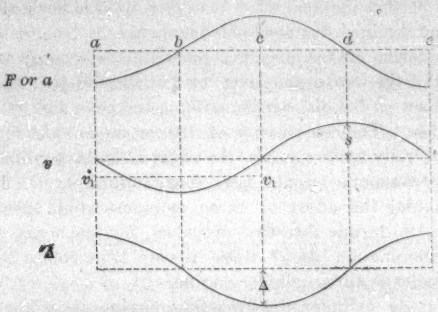


FIG. 1.

ity, and at c the maximum displacement occurs, for there the speed again becomes greater than average.

If the maximum displacement from mean is Δ, then during this time the change is from $+\Delta$ to $-\Delta$ or 2Δ for an average change in speed of $2\,s/\pi$.

That is, if speed is an average of $2\,s/\pi$ low during this time, the number of degrees passed will be $\theta\,(1 - 2\,s/\pi)$ and the change will be $2\,\Delta = \theta \times 2\,s/\pi$ or $\Delta = s\,\theta/\pi$, where $\theta = 360/2\,n$ approximately.

a the electrical displacement will be:

$$a = \frac{s\,p\,\theta}{\pi} \text{ or } \frac{s\,p\,360}{2\,\pi\,n} \text{ and } s = .0174\,\frac{a\,n}{p}$$

where p = number of pairs of poles

n = number of impulses per revolution.

It is not always safe to assume $\theta = 360/2n$, as in cross-compound engines one impulse is frequently very much suppressed (due to reciprocating parts) and another prolonged and enlarged, and the longer an unbalanced torque lasts, the greater the displacement even if the total unbalanced energy is no greater.

It is enteresting to note in this formula for a that the greater P, the greater the displacement angle and the greater n, the less the displacement. Hence the advantages of low frequency and multi-crank engines.

Substituting for a the value 2.5° which we have assumed, we find the allowable speed variation to be $s = .0435\ n/p$ where s is per cent. variation from mean.

A very good approximation of the fly-wheel necessary may be obtained quickly, thus:

The change of energy stored in fly-wheel for any change in speed is:

$$P_1 = w/2g \times (v_2^2 - v_1^2).$$

If v_0 is the mean linear velocity of w and $2s = (v_2 - v_1)/v_0$, then $v_2^2 - v_1^2 = v_0^2 \times 4s$, thus:

$$P_1 = w/2g \times v_0^2 \times 4s.$$

P_1 is the unbalanced energy of one impulse in foot pounds given by area P_1 in Fig. 3 and k, the unbalancing factor, is the ratio of this excess area to the energy of one revolution given by area enclosed by line of average torque.

Hence:

$$P_1 = k \times P = k \times \frac{\text{K.-W.} \times 33{,}000}{.746 \times eff \times \text{R. P. M.}}$$

where P is the energy per revolution of crank.

K.-W. = output of generator.

eff = efficiency of generator.

Equating these two values of P_1

$$\frac{w}{2g} \times v_0^2 \times 4s = k \times \frac{\text{K.-W.} \times 33{,}000}{.746 \times eff \times \text{R. P. M.}}$$

and putting for s its value in terms of displacement a

$$w\, v_0^2\, a\, n \times eff \times \text{R. P. M.} = k \times \text{K.-W.} \times p \times 41 \times 10^6$$

$$w\, v_0^2\, a \times eff \times \text{R. P. M.} = k \times \text{K.-W.} + p \times \theta \times 226 \times 10^3.$$

In this we have three quantities—w, weight of fly-wheel; k, unbalancing factor, and a, displacement—two of which may be varied to obtain a satisfactory value of the third.

The exact calculations are much longer and involve integrating first the torque curve (which is also an acceleration curve to another scale since $a = F/I$ to get a velocity curve and integrating this velocity curve to get the space curve.

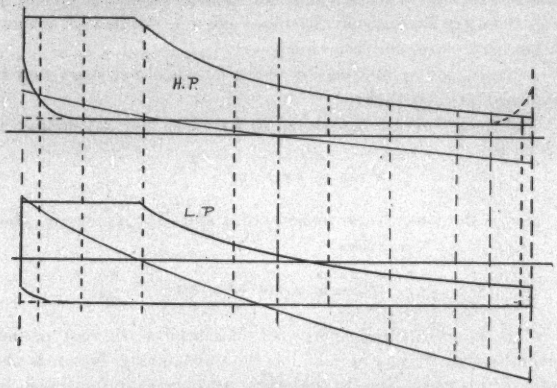


FIG. 2.

Let us take the two indicator cards shown in Fig. 2. These are the theoretical cards of a vertical cross-compound engine of 600-horse power at 150 R. P. M. to be direct connected to a three-phase 400-K.-W., 48-pole, 60-cycle alternator to be run in parallel with other generators driven by water-wheels. These cards and data were submitted by one of our largest engine building companies with their proposition in answer to a request for bids.

The cylinders are 17″ and 36″ in diameter, stroke 30″, connecting rod 75″ or 5 cranks, boiler pressure 150 lbs., vacuum 26″, 145 lbs. initial pressure in cylinder, and 22% cut-off in H. P. cylinder. Both cards are reduced to L. P. piston area and the spring scale is 30 lbs. per inch.

Weight of piston	284 H. P.	1591 L. P.
" " piston rod	178 "	178 "
" " cross-head	284 "	284 "
" " connecting rod	571 "	571 "

The fly-wheel weighs 34000 lbs. and the outside diameter is 12′, radius of gyration 4.67′.

TABLE I.

HIGH PRESSURE CYLINDER.

A Crank Angle.	B Stroke.	C Steam Pressure.	D Weight Parts.	E Inertia.	F Net Pressure.	H Sin ϕ.	I x.	K Tang. Effort.	L Moment.
....	....	28.8	+.7	—11.1	18.4	....	....	....	.....
18	.03	28.8		—10.5	19	.309	.368	7	8,900
36	.115	28.8		— 7.8	21.7	.588	.687	14.9	19,000
54	.237	26.4		— 4.8	22.3	.809	.905	20.2	25,800
72	.415	14.1		— .	14.2	.951	1.01	14.3	18,200
90	.600	7.5		+ 1.8	10	1.00	1.00	10	12,750
108	.725	5.1		+ 3.3	9.1	.951	.89	8.1	10,300
126	.833	3.6		+ 5.1	9.4	.809	.712	6.7	8,550
144	.922	2.4		+ 6.0	9.1	.588	.49	4.5	5,750
162	.980	—4.5		+ 6.6	2.8	.308	.248	1.6	2,040
180	1.00	—12.9		+ 7.5	—4.7	....	....	....	.....
LOW PRESSURE CYLINDER.									
90		29.5	+2.29	—32.	— .2		....	....	.....
108		29.5	2.3	—25.8	+ 6		.368	20.2	2,800
126		29.5		—18.9	12.9		.687	8.9	11,300
144		27.6		—11.7	18.2		.905	16.5	21,000
162		17.4		— 2.1	17.6		1.01	17.8	22,700
180		11.4		+ 6	19.7		1.00	19.7	25,100
198		9.6		+12	23.9		.89	21.3	27,200
216		7.5		+15.6	25.4		.712	18.1	23,100
234		6.6		+18	26.9		.49	13.2	16,800
252		3.6		+20	25.9		.248	6.4	8,200
270		....		+21.3	23.6		....	....	.....

Dividing the crank circle into a number of equal parts, say 20, each division representing 18°, we now find the position of piston when the crank passes each division; this is usually done graphi-

cally by striking circles of a radius $= l =$ connecting rod through each point on circle; this divides the diameter in a ratio equal to per cent. of stroke.

Analytically we get this[1] as

$$s = r\,(1 - \cos\varphi) + l\,[1 - \sqrt{1 - r^2 \sin^2\varphi/l^2}]$$

where:

$s =$ stroke in feet.
$r =$ crank length in feet.
$l =$ connecting rod length in feet.
$\varphi =$ crank angle assumed.

The net pressure on piston (difference in pressure on two sides) for each value of s is taken from indicator card and tabulated in Column C, Table 1. For convenience, pressure per square inch of low pressure cylinder is used.

The total weight of reciprocating parts divided by area of piston ($A = 1020$) is given in Column D. These are added on down stroke and subtracted on up stroke. In this instance only the down stroke is considered.

The inertia of the reciprocating parts is next taken as in Column E. This[2] is found from:

$$F = \frac{w\,v^2}{g\,r}\,(1 + r/l) \text{ and } F = \frac{w\,v^2}{g\,r}\,(1 - r/l)$$

the accelerating force at the beginning and end of the stroke respectively. In this $w =$ weight of piston, piston-rod, cross-head and half the weight of connecting rod, $g = 32$, $v =$ linear velocity of crank pin, $r =$ crank radius, $l =$ length of connecting rod.

The correct curve of inertia is very complicated, but is best approximated by plotting the two values given above and the point at which the acceleration is 0 and drawing a curve. This zero point is accurately enough given by $\tan\varphi = l/r$. This is given in Column E.

The acceleration of connecting rod used above differs very slightly from the correct value which is very complex, due to its peculiar motion.

Divide F by area of piston, and we have f, the pressure per square inch on piston.

1. Perry "The Steam Engine."
2. Unwin, "Machine Design," p. 68.

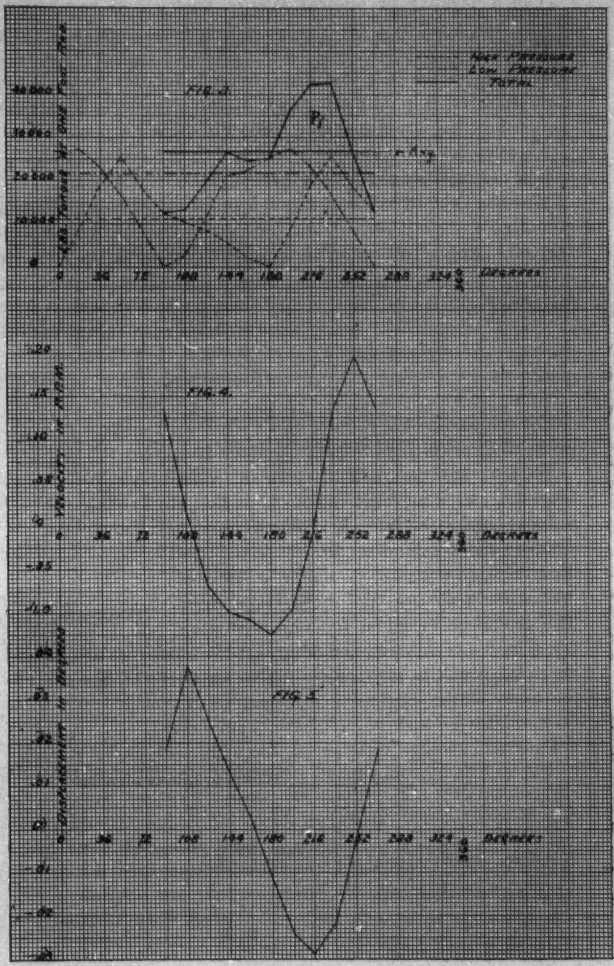


FIG. 3.
FIG. 4.
FIG. 5.
DEGREES

This is negative, or absorbs energy during the first half of stroke, and positive in the second half.

On the high pressure cylinder the effect of inertia is not very noticeable, but on the low pressure cylinder, due to its larger size, the effect is quite marked. It almost completely suppresses the peak of the impulse curve. (See Fig. 3).

These values in Columns C, D and E are combined algebraically, giving net pressure per square inch of piston, in Column F.

This has to be transferred to crank pin through an angle φ and would give $P \sin \varphi$ were it not for the shortness of the connecting rod. This effect causes the resolution of forces to be more complicated, and we have[3] the tangential effort T on crank. Column K.

$$T = P \sin \varphi \left(1 + \frac{r \cos \varphi}{\sqrt{l^2 - r^2 \sin^2 \varphi}}\right)$$

For comparison the value of this with $P = 1$ and $l/r = 5$, are given along with that of $\sin \varphi$, Columns H and I.

We have now the crank effort which, by multiplying by A, area of piston, and r the crank radius, gives the actual moment $F \times r$ in foot pounds. Column L. This we must also find for the L. P. cylinder and combine in proper phase relation, usually 90° behind the high pressure. Column A, Table II.

The next step is to find the average of the resultant moment. This should be averaged for one revolution, but if we take it for a half a revolution, assuming that the energy of two halves are equal, it will be satisfactory. This assumes that the effort is the same at the beginning and end of the half revolution. Column B is unbalanced moment at each period.

Find the average unbalanced force for each period (Column C, Table II) this acting on the fly-wheel produces an acceleration.

$F = Ia$

$a = Fg/w\, r^2$

in radians per sec. per sec.

where a is acceleration.

w = weight fly-wheel,

r = radius gyration.

3 Ewing, "The Steam Engine," p. 310.

As the speed does not vary to a great extent, we can assume without much error that equal angles are passed in equal times, therefore the time t in passing 18°, or 1/20 revolution

$$t = \frac{60 \text{ sec.}}{\text{R. P. M.} \times 20}$$

and at the end of the period the velocity will be changed $a \times t$ in radians per sec. or multiplying by $60/2\pi$ we get the change in R. P. M., Column D, Table II.

These values are the change in turns per minute for each period so at any point the actual velocity will be the progressive sums of these preceding gains $+ c$ (the velocity at the beginning of the stroke) since $\int dv = at + c$.

To find the velocity at the beginning of the stroke we sum all the values in Column E and get:

$$+ .062 - 1.1415 + 10c = 0$$

This is equal to 0, since we assumed 180° to be a complete period beginning and ending with the same point, hence all the positive values must equal all the negative values. Having found c, we add this to each value in Column E, and get the actual velocity in Column F.

We must now integrate the velocity curve to get displacement curve. This is done in the same way as we obtained velocity from acceleration, thus: Find average velocity for each period. Multiply the average velocity for each period by

$$t = \frac{1}{150 \text{ x } 20}$$

the time in minutes to pass 18°, and we get the displacement in turns, and multiply this by 360 and get the displacement in degrees of the crank circle. This is given in Column H as change of position. Summing these as we did the acceleration, we get $(\Delta + c)$, the relative position.

Eliminating c we get in Column K the final displacement from mean on crank circle.

To find the electrical displacement select the maximum value and multiply by p, the number of pairs of poles on the alternator.

This is: $.0379 \times 24 = .91°$

TABLE II.

Crank Angle.	A Total Torque.	B Unbalanced Torque.	C Average Excess.	D Gain in R. P. M.	E Relative Velocity at +C.	F Velocity.	G Average Velocity.	H Change of Position Degrees.	Relative Position.	Displacement.
90	12750	—14200	—14025	.....	0	+.135	+.077	.....	0	+.0187
108	13100	—13850	—13475	—.116	—.116+C	+.019	—.025	+.0092	+.0092+C	+.0379
126	19850	—7100	—3650	—.087	—.203+C	—.068	—.083	—.0090	+.0002+C	+.0249
144	26750	— 800	— 1205	—.03	—.233+C	—.098	—.103	—.010	—.0088+C	+.0189
162	24740	+ 2210	— 2030	—.01	—.243+C	—.108	—.116	—.0124	—.0162+C	+.0025
180	25100	+ 1850	— 3650	—.017	—.260+C	—.125	—.110	—.0139	—.0301+C	—.0114
198	36100	+ 9150	+ 03	+ 03	—.230+C	—.095	—.044	—.0132	—.0433+C	—.0246
216	42100	+15150	+ 12150	+ 101	—.129+C	+.006	+.070	—.0053	—.0486+C	—.0089
234	41600	+15650	+15400	+.128	—.001+L	+.134	+.165	+.0074	—.0412+C	—.0015
262	26400	— 550	+ 7350	+.063	+.062+C	+.197	+.166	+.0198	—.0824+C	—.0017
270	12750	—14200	— 7350	+.061		+.135		+.0191	.0005+C	+.0187

This engine has a very small unbalancing factor and large fly-wheel, and is a very good one for the purpose. The fly-wheel could be made lighter. I might say that these particular engine builders claim to make a point of using a safety factor of 50%, at least in meeting our requirements.

A paper presented at the 157th Meeting of the American Institute of Electrical Engineers, New York, October 25th, 1901. President Steinmetz in the Chair.

PARALLEL RUNNING OF ALTERNATORS.

BY ERNEST J. BERG.

In the following article an endeavor will be made to show the principles govering parallel operation of alternators, driven by steam engines or turbines. The laws of parallel running when generators are driven by electric motors depend so much upon the action of the motors that this part of the subject is omitted and reserved for another paper.

Consider two alternators intended for parallel operation, and let:

e = induced E. M. F.

$Z_1 = r_1 - j\,x_1$

$Z_2 = r_2 - j\,x_2$ = the internal impedance of the different armatures.

Y_1 and Y_2 = the corresponding admittances.

$Z = r - j\,x$ = the impedance of the external load.

Y_3 = joint admittance of the armature circuits.

Z_3 = corresponding impedance.

Z_0 = total impedance in the circuit.

We have then:

$$\left.\begin{aligned} Y_1 &= \frac{r_1}{Z_1^2} + j\,\frac{x_1}{Z_1^2} \\ Y_2 &= \frac{r_2}{Z_2^2} + j\,\frac{x_2}{Z_2^2} \end{aligned}\right\} \quad Y_3 = Y_1 + Y_2$$

$$Z_3 = \frac{1}{Y_3}$$

$$Z_0 = Z_3 + Z$$

$$\text{Total current } I = \frac{e}{Z_3 + Z}$$

$$\text{Voltage across load} = I\,Z = \frac{e\,Z}{Z_3 + Z}$$

$$\text{Drop in armatures} = e - \frac{e\,Z}{Z_3 + Z}$$

$$\text{Current in armature No. 1} = \left(e - \frac{e\,Z}{Z_3 + Z}\right) Y_1$$

$$\text{Current in armature No. 2} = \left(e - \frac{e\,Z}{Z_3 + Z}\right) Y_2$$

$$\text{Volt-amperes of one armature} = e^2\,Y_1\left(1 - \frac{Z}{Z_3 + Z}\right)$$

$$\text{Volt-amperes of one armature} = e^2\,Y_2\left(1 - \frac{Z}{Z_3 + Z}\right)$$

Power of each armature is the real part of the expression for volt-amperes.

This is expressed as:

$$P_1 = e^2\,\frac{(p\,r_1 - q\,x_1)}{(m^2 + n^2)(r_1^2 + x_1^2)}$$

and

$$P_2 = e^2\,\frac{(p\,r_2 - q\,x_2)}{(m^2 + n^2)(r_2^2 + x_2^2}$$

where:

$$p = m^2 + n^2 - m\,r - n\,x$$

$$q = m\,x - n\,r$$

$$m = \frac{G}{G^2 + B^2} + r$$

$$n = \frac{B}{G^2 + B^2} + x$$

$$G = g_1 + g_2$$
$$B = b_1 + b_2$$
$$Y_1 = g_1 + j\, b_1$$
$$Y_2 = g_2 + j\, b_2$$

Neglecting the resistances of the armatures as compared with the reactances, we get the following expressions:

$$P_1 = \frac{e^2 x_2 r (x_1 + x_2)}{Z^2 (x_1 + x_2)^2 + x_1^2 x_2^2 + 2 w x_1 x_2 (x_1 + x_2)}$$

$$P_2 = \frac{e^2 x_1 r (x_1 + x_2)}{Z^2 (x_1 + x_2)^2 + x_1^2 x_2^2 + 2 w x_1 x_2 (x_1 + x_2)}$$

In other words, the alternators divide the load inversely as their armature reaction.

With a non-inductive external circuit, these questions become:

$$P_1 = \text{power of one armature} = \frac{e^2 r x_2 (x_1 + x_2)}{r^2 (x_1 + x_2)^2 + x_1^2 x_2^2}$$

$$P_2 = \text{power of other armature} = \frac{e^2 r x_1 (x_1 + x_2)}{r^2 (x_1 + x_2)^2 + x_1^2 x_2^2}$$

which with close approximation can be written:

$$P_1 = \frac{e^2 x_2}{r (x_1 + x_2)}$$

$$P_2 = \frac{e^2 x_1}{r (x_1 + x_2)}$$

Therefore two machines of same induced E. M. F. and armature reaction running in phase will share the load evenly. If one has twice as high armature reaction as the other, it will take one half of the load, etc. Thus since machines of identical regulation have practically armature reactions inversely proportional to their size, it is possible to run such machines from the same engines at proper share of the total load. Should, however, two machines of the same size and different armature reaction run from the same engine, the distribution of power will not be in accordance with the rating.

To balance the load it is then necessary to add reactance in one armature or to mechanically offset one armature in regard to the other, or, finally, to run them at such field excitation as will cause considerable cross-current, which effect will be discussed later.

In the discussion given above it was assumed that the machines were running strictly in phase, which would practically limit the case to alternators clutched together. If, however, they are rigidly connected together, but out of phase, the current which is due to the resultant of the two E. M. F.'s has a different phase displacement from the E. M. F. of one machine than it has from that of the other, that is, the energy corresponding is different in the two machines, or the one supplies more power than the other.

The relation of currents to E. M. F.'s can also be changed by changing the excitation.

Assume machines of the same armature reaction which, before being thrown in parallel, have unequal E. M. F.'s. After being connected together their terminal voltages must be the same, therefore a cross-current must flow between them which, in the higher excited alternator, is lagging, and in the other leading; which currents, in conjunction with the field excitation, give a resultant field of same magnitude in the two, but displaced in phase. In the higher excited machine it will bring the resultant field more in phase with the energy current, thus giving a result equivalent to an armature of lower reactance; in the lower excited machine a result same as higher armature reaction. It is therefore, possible to change the load by change of field excitation by sacrifice of current, since this method involves wattless currents between the generators.

In view of the above discussion, it is thus evident that the question of distribution of power between alternators is quite simple, if they are running at absolutely the same speed at all loads, that is, if the engines regulate so closely as to permit the clutch to be opened and the armatures still retain the same positions. In reality, however, engines cannot be made to govern so closely, but one will tend to hold up the speed somewhat better than the other. That being the case, it is safe to say that as a rule the engines will not tend to give the same power at exactly the same speed.

Referring back to the equations and discussions we find, there-

fore, that with the same armature reaction and induced E. M. F., if different power is given each machine, the armatures must be offset a certain and fixed degree, which means that by even the most minute difference in speed regulation the armatures are forced to change their relative positions, which obviously is a cause for "hunting."

The relative power given by each machine depends upon the respective powers of the engines at identical speeds.

From the above discussion it is apparent that a definite tendency for "hunting" exists, and that this hunting is manifest in a mechanical pulsation in the speed of the armatures, and a transfer of power from one unit to another.

A most superficial investigation will show that this flow of power is much greater than that corresponding to the stored energy of the armature fly-wheel, etc., at the relatively small differences in speed, and that pulsating power actually has to be supplied by the engines. Therefore, and since power cannot be obtained from an engine without corresponding steam, the flow of steam must be periodic to sustain an accumulated hunting.

The remedy is therefore to be found by preventing such periodic flow of steam, which can be accomplished by proper dash pot arrangements at the governor. Other remedies also suggest themselves, the favorite being to make a short-circuited winding in and around the field of alternators. The object of such winding obviously is to establish a load when these oscillations occur, which load tends to stop the acceleration and retardation of the armature. The disadvantages of this method are that losses of no small magnitude not only are introduced when the machines hunt, but also when they are running at steady speed, which losses are caused by induced currents in this short-circuited winding, due to pulsations of flux incidental to modern construction of iron clad armatures.

Schenectady, October 14th, 1901.

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Total 54

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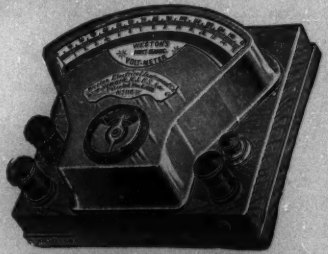

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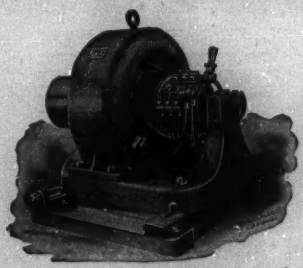

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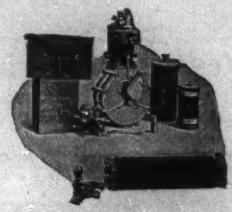

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